ESTIMATION ERRORS AND TIME VARYING BETAS IN EVENT STUDIES
A NEW APPROACH

Jayanth Rama Varma and Samir K Barua

Working Paper 759

July 1988

Indian Institute of Management
Ahmedabad
ESTIMATION ERRORS AND TIME VARYING BETAS IN EVENT STUDIES
A NEW APPROACH

Jayanth Rama Varma and Samir K Barua

ABSTRACT

The event study is one of the most powerful techniques for studying market efficiency. Over a period of time, researchers have made several modifications to the original methodology of Fama, Fisher, Jensen and Roll (1969). Nevertheless, the current methodology continues to suffer from several grave deficiencies. These deficiencies arise due to (a) a failure to take into account the variance covariance structure of the estimated abnormal returns (across time and across securities) and (b) fundamental shortcomings of the moving window technique used to deal with possible changes in the betas in the neighbourhood of the event. Our proposed methodology overcomes these deficiencies and provides statistically efficient estimates. We then extend the analysis to handle nonstationary parameters evolving according to a Kalman Filter model.
0. Introduction

Event studies seek to analyse the impact of a specified class of events on the prices of securities. The most widespread use of the event study is in testing the Efficient Market Hypothesis (EMH). Efficiency is demonstrated by showing that the market response to an event takes place either before the event or very shortly after the event - information is either anticipated or very quickly assimilated. The pioneering work on event study was done by Ball and Brown (1968) and Fama, Fisher, Jensen and Roll (1969) (henceforth referred to as FFJR). The methodologies used in these studies have become a standard technique for testing the EMH. Over the last two decades a variety of events such as announcement of stock splits, announcement of earnings, mergers and takeovers have been studied by researchers for examining market efficiency. Though several modifications have been made to the original methodologies, their basic structure has remained unaltered. In this paper, we first review the development of the event study methodology and conclude that it still suffers from several grave deficiencies, which come in the way of proper testing of the EMH. We then propose alternative methodologies to overcome the limitations identified.

By now it is well recognised that any event study to test the EMH is necessarily a joint test of the model for explaining the return structure of securities and the EMH. There appears to be no way of eliminating the impact of a mis-specification of the model from the testing procedure. In some event studies, such as announcement of earnings, where the event itself is defined on the basis of an expectation model, the situation is even worse as it becomes a joint testing of the expectation model for earnings, the model for explaining return structure of securities and the EMH. We do not offer any solution to this major limitation. We focus on the rest of the event study methodology which, we believe needs improvement.
1. The FFJR Methodology

The FFJR approach begins by specifying the market model for each security in the following form:

\[ R_{jt} = \alpha + \beta_j R_{Mt}^j + u_{jt} \]  

where for security \( j \) and time \( t \), \( R_{jt} \) is the logarithm of the price relative, adjusted for dividends; \( R_{Mt}^j \) is the logarithm of the market index relative and \( u_{jt} \) is the error term which has mean zero.

The subscript \( t \) for time has a special meaning. It is equal to zero for the period in which the event (the announcement of stock split in their case) takes place; negative values of \( t \) indicate the pre-event period while positive values indicate the post-event period.

Having interpreted time in the above manner, they make a preliminary assessment about the period around the event (say \( -r \leq t \leq s \) ) in which the returns may behave abnormally. The coefficient of the market model, \( \alpha_j \) and \( \beta_j \) are then estimated through OLS, using the data from the stable periods \( t < -r \) and \( t > s \). The market model, thus estimated, is then used to compute the error terms for periods around the event:

\[ \hat{u}_{jt} = \hat{R}_{jt} - \hat{\alpha}_j - \hat{\beta}_j \hat{R}_{Mt}^j \]  

To capture the overall market response to announcement of stock splits, the following aggregation scheme is proposed:

\[ \hat{\alpha}_t = \frac{\sum_{j=1}^{m(t)} \hat{u}_{jt}}{m(t)} \]  

\[ \hat{A}_t = \sum_{t=-r}^{t} \hat{\alpha}_t \]  

\[ \hat{A}_t = \sum_{t=-r}^{t} \hat{\alpha}_t \]  

\[ \hat{A}_t = \sum_{t=-r}^{t} \hat{\alpha}_t \]
where \( m(t) \) is the number of securities on which observations are available in period \( t \).

The behaviour of \( \hat{a}_t \) and \( \hat{A}_t \), the average residuals and the cumulative average residuals, are examined to make inferences about market efficiency. FFJR found that \( \hat{a}_t \) was continuously positive during the pre-event period, leading to a continuous high rate of increase in the value of \( \hat{A}_t \) up to the event period. However, the rate of increase in the value of \( \hat{A}_t \) becomes almost zero immediately after the event period.

They therefore concluded that the market is efficient. They refined their analysis by classifying the securities into two groups: one comprising those that announced higher dividends following stock split and the other comprising scrips of companies that announced a cut in dividends. They concluded that unusually high returns in the months immediately preceding the month in which stock split is announced reflects the market's anticipation of substantial increases in dividends, which, in fact occur.

2. The Modifications to the FFJR Methodology

Later studies have made several modifications to the FFJR methodology outlined above. These modifications attempt to eliminate some of the limitations of the FFJR approach. The modifications made are:

a. Return Generating Process

Researchers have used a variety of models of the return generating process in lieu of the market model employed by FFJR; these include various versions of the CAPM and the two factor model of Fama-McBeth (1973). They compute the abnormal returns on the basis of the assumed return generating process.

After extensive testing of several models such as mean adjusted returns, market adjusted returns, market and risk adjusted returns, Brown and Warner (1980) conclude that when events are not clustered in time, the differences between the various methodologies are quite small. For the case where events
are "clustered", the mean adjusted returns model is decidedly inferior. In either case, there is little to choose between different methods which adjust for market factors. Their final conclusion is: "A bottom line that emerges from our study is this: beyond a simple, one factor market model, there is no evidence that more complicated methodologies convey any benefit." Brown & Warner (1985) and Thompson (1988) provide further support for this conclusion.

b. Time Varying Betas

Researchers had noticed that a security appears to be far more volatile close to the event period. This led to a suspicion that the beta of a security may be undergoing changes during the "unstable" period close to the event date. In such a case, using a constant beta estimated using data from the stable period may not be valid. Bar-Yosef and Brown (1977) confirmed this suspicion by estimating betas around the event by using a "moving window" approach. (In this approach, the sample chosen for estimating beta for any period consists of a small block of observations close to the period). Using a symmetric window for estimating beta for each period, they concluded that the beta rises prior to a stock split, reaches a peak near the split, and drops back again to normal level after the stock split. This "moving window" approach in which different betas are used in different periods has been employed by several researchers such as Charest (1978), Thompson (1978), Watts (1978) and Malatesta (1983) in event studies. The average residuals and the cumulative average residuals are computed exactly the way FFJR had done in their study.

c. The Market Response to the Event

The reason for computing $\hat{\alpha}_t$ and $\hat{\Delta}_t$ is primarily to examine the aggregate market response to an event. Several aggregation schemes, in addition to the FFJR scheme, based on arithmetic average have been used by researchers. The portfolio test approach has been used by Mandelker (1974), Jaffe (1974), Ellert (1976), Charest (1978) and Malatesta (1983) among several others. The approach begins by specifying a rule for forming a portfolio for trading in the market. The portfolio's residual or abnormal performance is then used to test the EMH. A variety of rules for forming portfolios have been used.
d. Testing Significance

FFJR did not test the significance of \( \hat{\alpha}_t \) and \( \hat{A}_t \); they merely examined their behaviour around the event date. Hence, they were not concerned about the sampling variances of these statistics. The subsequent works which tested the significance were required to estimate these variances. All the researchers have estimated these variances under the simplest possible assumptions ignoring both cross-sectional as well as serial correlation. The standard deviation calculated from a time series of security or portfolio residuals is used to provide an estimate of the standard error for the t-test.

3. Critical Analysis of the Current Methodology

The most remarkable feature of the published work on event studies is the absence of a clear statement of the model being used and the hypothesis being tested. These are possibly perceived as too obvious to need an explicit statement. We however find that a clear statement of the model and the hypothesis is essential for a proper discussion of the methodology. The FFJR methodology implicitly postulates the following model:

\[
R_{jt} = \alpha_j + \beta_j R_{Mt} + u_{jt} \quad t < -r; t > s \quad (5)
\]

\[
R_{jt} = (\alpha_j + a_t) + \beta_j R_{Mt} + u_{jt} \quad -r \leq t \leq s \quad (6)
\]

The hypothesis to be tested is,

\[
H_0 : a_t = 0 \quad \text{for} \ 0 < t \leq s \ \text{(market is efficient)}
\]

OR

\[
: a_t = 0 \quad \text{for} \ -r \leq t \leq s \ \text{(event has no impact on the market)}
\]

The test statistics used for testing the hypothesis are the average(\( \hat{\alpha}_t \)) and the cumulative average(\( \hat{A}_t \)) residuals, as discussed earlier. The time points \(-r\) and \(s\) define the period around the occurrence of the event when the security is expected to show abnormal returns.

The recognition that betas change from period to period during the "unstable" period implies that in the above models \( \beta_j \)
would have to be replaced by $\beta_{jt}$. The hypothesis to be tested would remain the same. The betas would have to be estimated separately for each period.

Given the above statement of the models and the hypothesis to be tested, the following comments can be made on the current methodology:

a. The aggregate market response in period $t$ ($a_t$) is estimated using a simple arithmetic average of $\hat{u}_{jt}$'s. Such a procedure implicitly assumes that the assumptions needed for OLS are valid. We shall show below that the variance structure of $\hat{u}_{jt}$'s violates the OLS assumptions in several respects. Hence, a more elaborate (GLS) procedure is essential for estimating $a_t$ from $\hat{u}_{jt}$'s.

b. The test of significance of departure from the null hypothesis is imperative for any study on EMH. However, in assessing the variance of the test statistics, none of the studies recognize the variance structure of $\hat{u}_{jt}$'s. Even the portfolio approach which has become a very popular procedure for assessing aggregate market response suffers from the same deficiency. A correct assessment of variances, after accounting for the interrelationships among $\hat{u}_{jt}$'s could lead to very different inferences in empirical work.

c. The moving window approach for estimating betas is erroneous because the assumption that $E(u_{jt})=0$ is unlikely to be true near the event period even if the market is efficient. In such a case, the estimated betas would be biased. In addition, whenever the sample used for estimating beta for a period straddles the period, the observation used for estimation of beta is again used for assessing error in prediction ($\hat{u}_{jt}$). This would bias the values of $\hat{u}_{jt}$'s. Besides, there appears to be no justification for using future returns for estimating current period's betas, when the assumption is that the betas are changing during that time span. (Investors could certainly not have used these future returns in forming their expectations of beta). In our view, therefore, the moving window approach has little theoretical justification. This does not mean that the difficulty because of changing betas is unimportant. But that difficulty must be tackled differently.
4. The Variance Structure of $\hat{u}_{jt}$'s

As we have stated earlier, the variance structure of $\hat{u}_{jt}$'s must be analysed both for estimating $a_t$ and for testing its significance. We, therefore, examine the $\hat{u}_{jt}$'s for heteroscedasticity, serial-correlation and cross-correlation. It is important to recognize that the $\hat{u}_{jt}$'s are residuals from an estimated regression line and not the true regression line. Unlike the $u_{jt}$'s they are therefore contaminated by estimation errors. As a result, the variance structure of $\hat{u}_{jt}$'s is much more complex than that of $u_{jt}$'s.

a. Heteroscedasticity

It is unlikely that all securities would have identical error variance, $\text{Var}(u_{jt})$. Therefore, a correction for cross sectional differences in error variances is imperative. But what is noteworthy is that even if the securities had identical error variances, a correction for heteroscedasticity would still be needed because of estimation errors, as shown below.

\[
\hat{u}_{jt} = R_{jt} - \alpha - \beta R_{jt}
\]

\[
= (\alpha + a + \beta R_{jt} + u_{jt}) - \alpha - \beta R_{jt}
\]

\[
= a + (\alpha - \alpha) + (\beta - \beta)R_{jt} + u_{jt} \quad -r \leq t \leq s
\]

\[
\text{Var}(\hat{u}_{jt}) = \text{Var}(\hat{\alpha}) + \text{Var}(\hat{\beta})R_{jt}^2 + 2 \text{Cov}(\hat{\alpha}, \hat{\beta})R_{jt} + \sigma_{jt}^2
\]

where $\sigma_{jt}^2 = \text{Var}(u_{jt})$
b. Serial-correlation

While estimating the market model through OLS, the implicit assumption is that $\text{Cov}(u_{jt}, u_{jt}) = 0$. However, this does not imply that $\text{Cov}(\hat{u}_{jt}, \hat{u}_{jt}) = 0$. In fact, because of estimation errors, the covariance between any two errors of prediction would be:

$$\text{Cov}(\hat{u}_{jt}, \hat{u}_{jt}) = \text{Var}(\hat{\alpha}_j) + \text{Var}(\hat{\beta}_j) R_{jt} R_{jt}$$

$$+ \text{Cov}(\hat{\alpha}_j, \hat{\beta}_j) (R_{jt} + R_{jt})$$

$$j \quad j \quad Mjt \quad Mjt$$

c. Cross-correlation

Researchers have argued that since the calendar time periods under consideration for various securities are likely to be non-overlapping, cross-sectional correlation among $\hat{u}_{jt}$'s may be assumed to be zero. This argument is valid when applied to the $u_{jt}$'s and we can conclude that $\text{Cov}(u_{jt}, u_{kt}) = 0$. Let us then write the expression for $\text{Cov}(\hat{u}_{jt}, \hat{u}_{kt})$.

$$\text{Cov}(\hat{u}_{jt}, \hat{u}_{kt}) = \text{Cov}(\hat{\alpha}_j, \hat{\alpha}_k) + \text{Cov}(\hat{\beta}_j, \hat{\beta}_k) R_{jt} R_{kt}$$

$$+ \text{Cov}(\hat{\alpha}_j, \hat{\beta}_k) R_{jt} + \text{Cov}(\hat{\alpha}_k, \hat{\beta}_j) R_{kt}$$

$$j \quad j \quad Mjt \quad Mkt \quad k \quad k \quad Mjt$$

Since $\alpha_j$, $\beta_j$ and $\alpha_k$, $\beta_k$ are coefficients estimated from two different regression specifications satisfying $\text{Cov}(u_{jt}, u_{kt}) = 0$, it can be shown that all the covariances on the RHS of the above expression are zero and $\text{Cov}(\hat{u}_{jt}, \hat{u}_{kt})$ is also zero.

5. The Proposed Methodology

We find it necessary to distinguish three alternative sets of assumptions about the behavior of the alphas and betas:
1. Alphas and betas are stationary except for possible abnormal alphas around the event date. This corresponds to the assumptions of the original FFJR study.

2. Alphas and betas are stationary except for an abnormal period around the event date in which there may be both abnormal alphas and abnormal betas. Evidence for abnormal betas around the stock split was provided by Bar-Yousef & Brown (1977).

3. Alphas and betas may be nonstationary even in the non-event period. Conclusive empirical evidence exists for beta nonstationarity (Fabozzi & Francis (1978), Roenfeldt et al (1978), Hsu(1982), and Ohlson & Rosenberg (1982) ) and it is essential to take this into account.

As usual, increased realism is obtained at the price of increased econometric complexity. The detailed methodology for the three cases is presented below.

a. Stationary Betas

In this case, the model is as stated in (5) & (6). From the discussion above on the presence of heteroscedasticity and serial correlation it is clear that the OLS procedure would be inefficient for estimating the $a_t$. In fact, efficient estimation of all $a_t$ would require use of a single GLS procedure, similar to Zellner's (1962) Seemingly Unrelated Regression (SUR) model (see Theil (1971)). This is because the $\hat{\alpha}_t$ for different time periods are correlated, as shown above. The GLS/SUR procedure is certainly feasible given modern computing technology. However, this procedure may be avoided if we have a sufficiently large sample size so that the estimation errors in alphas and betas are quite small. It would even then be necessary to use GLS procedure for estimating each $a_t$ separately.

b. Model with Shift in Betas

If shifts in betas are allowed around the event period, the model specified in (6) would have to be replaced by the following specification:
\[ R = (\alpha + a) + (\beta + b) R + u \quad \text{for} \quad -r \leq t \leq s \quad (7) \]

In this model shifts in both alphas and betas are allowed. It is clear that if the true model is given by (7), the estimate of \( a_t \), using the earlier model would be biased because of the omitted variable. The extent of bias would be \( b_t \bar{R}_{Mt} \).

We shall modify the above specification slightly by subtracting the (continuously compounded) risk-free rate \( R_F \) from both \( R_{jt} \) and \( R_{Mt} \); this would eliminate the dependence between \( a_t \) and \( b_t \) that would otherwise exist because of the CAPM relationship. Once this is done, a non-zero \( b_t \) does not represent a departure from equilibrium, and has no implication for market efficiency. Denoting \( (R_{jt} - R_{Ft}) \) by \( r_{jt} \) and \( (R_{Mt} - R_{Ft}) \) by \( r_{Mt} \), we arrive at the specification:

\[ r = (\alpha + a) + (\beta + b) r + u \quad \text{for} \quad -r \leq t \leq s \quad (7') \]

To estimate \( a_t \) and \( b_t \), we rewrite (7') as:

\[ \hat{r} = \alpha + (\alpha - \alpha) + a + (\beta + \beta - \beta) + b \quad \text{for} \quad -r \leq t \leq s \quad (8) \]

OR

\[ \hat{u} = a + b \quad \text{for} \quad -r \leq t \leq s \quad (8) \]

where

\[ \hat{u} = r - \alpha - \beta \]

\[ \hat{v} = (\alpha - \alpha) + (\beta - \beta) + u \]

The residuals \( v_{jt} \) have mean zero. Their variances and covariances are the same as those of \( \hat{u}_{jt} \) derived earlier. This specification can again be estimated using GLS/SUR procedure. If a single GLS is computationally difficult and estimation errors are small, we may, as in the earlier case, use separate GLS regressions for each time period to estimate \( a_t \) and \( b_t \).
In case we allow time varying parameters in models (5), (6), and (7), then we could postulate a Kalman Filter model (see Chow (1984)) under which the alphas and betas evolve according to

\[
\begin{align*}
\alpha_j & = \alpha_j + \delta_{j,t-1} + \delta_{1,j,t} \\
\beta_j & = \beta_j + \delta_{j,t-1} + \delta_{2,j,t}
\end{align*}
\]  
\tag{9}

where \( \delta_{j,t} = (\delta_{1,j,t}, \delta_{2,j,t}) \sim N(0, \Omega_j) \)

Unlike in model (7), we do not restrict the changes in alphas and betas to the period around the event; nor do we require them to be cross-sectionally constant. Equations (5), (6), (7) and (8) would then be replaced by,

\[
\begin{align*}
\alpha_j & = \alpha_j + \beta_j \alpha_j + u_j & t < -r; t > s \tag{10} \\
\beta_j & = (\alpha_j + a) + \beta_j \alpha_j + u_j & -r \leq t \leq s \tag{11} \\
\gamma_j & = (\alpha_j + a) + (a + b) \alpha_j + u_j & -r \leq t \leq s \tag{12} \\
\end{align*}
\]

\[
\begin{align*}
\gamma_j & = a + b \alpha_j + v_j & -r \leq t \leq s \tag{13}
\end{align*}
\]

The analysis would then begin by estimating model (10) subject to eqn. (9). This estimation would use Kalman Filtering; the matrices \( j \) in (9) would be estimated by Maximum Likelihood (see Chow (1984) for details). For simplicity, we shall assume that data beyond \( s \) is not used. (To use data beyond \( s \), we would have to treat the data for \(-r \leq t \leq s\) as missing observations.)
while estimating model (10). Use of data beyond s will also complicate the derivations below as we will have to use a weighted average of the coefficients at -(r+1) and (s+1) to estimate the coefficients in between.

The next step would be to obtain the estimates of $\alpha_{jt}$ and $\beta_{jt}$ for $-r \leq t \leq s$ using data prior to $-r$ only. Eqn (9) implies that these estimates are simply $\hat{\alpha}_{jt,-(r+1)}$ and $\hat{\beta}_{jt,-(r+1)}$. The variances and covariances of $\hat{\alpha}_{jt}$ and $\hat{\beta}_{jt}$ are more complicated:

\[
\text{Var}(\hat{\alpha}_{jt}, \hat{\beta}_{jt}) = \text{Var}(\hat{\alpha}_{j,-(r+1)}, \hat{\beta}_{j,-(r+1)}) + (t+r+1)\Omega
\]

\[
\text{Cov}(\hat{\alpha}_{jt}, \hat{\alpha}_{jt}) = \text{Var}(\hat{\alpha}_{j,-(r+1)})
\]

\[
\text{Cov}(\hat{\beta}_{jt}, \hat{\beta}_{jt}) = \text{Var}(\hat{\beta}_{j,-(r+1)})
\]

\[
\text{Cov}(\hat{\alpha}_{jt}, \hat{\beta}_{jt}) = \text{Cov}(\hat{\alpha}_{j,-(r+1)}, \hat{\beta}_{j,-(r+1)})
\]

\[
\text{Cov}(\hat{\alpha}_{jt}, \hat{\alpha}_{jt}) = \text{Cov}(\hat{\beta}_{j,-(r+1)}, \hat{\beta}_{j,-(r+1)}) = 0 \quad j \neq k
\]

Using the above formulae, we can compute the variance-covariance matrix of $v_{jt}$ in (13) using a procedure similar to that described in section 4.

The final step would be to use GLS/SUR in eqn (13) to estimate $a_t$ and $b_t$. Unlike in the earlier cases, it is now quite unreasonable to ignore estimation errors in $\hat{\alpha}_{jt}$ and $\hat{\beta}_{jt}$. This is because, under nonstationarity, there are no consistent estimators of $\alpha_{jt}$ and $\beta_{jt}$. The full GLS/SUR procedure is mandatory.

In our view, the computational difficulty of the GLS
regression is not very great because residuals of different securities are uncorrelated. The variance covariance matrix is, therefore, block diagonal with blocks of dimension \((r+s+1) \times (r+s+1)\). In most event studies, \(r\) and \(s\) are quite small — usually around 10-20. Hence, the inversion of the matrix for GLS estimation is computationally quite feasible. The proposed methodology for event studies under time varying betas with both alpha and beta shocks is, therefore, statistically efficient and computationally practicable.

6. Conclusion

Our analysis traces the deficiencies of the current event study methodology to two main sources:

a. The market response to an event is estimated from the estimated abnormal returns \(\hat{u}_{jt}\) which are aggregated in some way and tested against the hypothesized value of zero. The \(\hat{u}_{jt}\) are, however, the residuals from an estimated regression line and are contaminated by estimation errors in alphas and betas. The \(\hat{u}_{jt}\), therefore, have a complex variance-covariance structure which exhibits marked heteroscedasticity and serial correlation. However, in current methodology, neither the aggregation schemes nor the testing procedures take this variance structure into account.

b. The moving window approach for handling shifts in the betas near the event date is fundamentally unsound. It estimates the parameters (alphas & betas) from an unstable period in which \(E(\hat{u}_{jt}) \neq 0\), even if the market is efficient. In some versions, the moving window uses data which no investor could have had in forming his expectations, and calculates residuals from a regression line which has been estimated using that observation itself.

We have argued that a proper event study methodology must be based on explicitly formulated models. Rather than use the ad-hoc device of moving windows we model the shift in betas by an explicit shift term \((b_t)\) to be estimated. Rather than use ad-hoc aggregation schemes, we model the market response as a shift term \((a_t)\) which can be estimated. Finally, we take the variance
structure of the $\hat{\mu}_j$ into account and present an efficient GLS estimation methodology.

We have finally extended the model to handle nonstationary parameters evolving according to a Kalman Filter model.

Despite all these refinements, we believe that our methodology remains computationally tractable.
REFERENCES


