Risk Management at Indian Exchanges

Going Beyond SPAN and VaR

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Where do we stand today?

- **Risk systems in exchange traded derivatives (ETD) were designed from a clean slate in 1990s.**
- **Drew on then global best practices – for example, Risk Metrics and SPAN.**
- **Many incremental improvements were made subsequently.**
- **But core foundations are a decade old.**

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What is the state of the art?

- **Academic risk measurement models today emphasize:**
  - **Expected shortfall and other coherent risk measures and not Value at Risk**
  - **Fat tailed distributions and not multivariate normal**
  - **Non linear dependence (copulas) and not correlations**

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Scaling Up

- **Risk Metrics and SPAN are highly scalable and proven models.**
- **Can new models scale up?**
  - **Moore's law over last 15 years enables thousand fold increase in computations**
  - **But curse of dimensionality must be addressed: computational complexity must be linear in number of portfolios, positions and underlyings: $O(n)$**

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L C Gupta Report: Value at Risk

- **"The concept of "value at risk" should be used in calculating required levels of initial margin. The initial margin should be large enough to cover the one-day loss that can be encountered on the position on 99% of the days."**

  L. C. Gupta Committee, 1998

  Paragraph 16.3(3)

- **99% VaR is the worst of the *best* 99% outcomes or the *best* of the 1% worst outcomes.**

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Value at Risk (VaR)

- **Why *best* of the worst and not average, worst or most likely of the worst?**
  - **Worst outcome is –∞ for any unbounded distribution.**
  - **VaR is mode of the worst outcomes unless hump in tail.**
  - **For normal distribution, average of the worst is**

    $n(VaR)/\mathcal{N}(VaR)$ **and is asymptotically the same as VaR because**

    $1 - \mathcal{N}(y) \sim n(y)/y$ **as** y **tends to** ∞

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Expected Shortfall

- For non normal distributions, VaR is not average of worst 1% outcomes. The average is a different risk measure – Expected Shortfall (ES).
- ES does not imply risk neutrality. Far enough in the tail, cost of over and under margining are comparable and the mean is solution of a quadratic loss problem.

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Coherent Risk Measures

- **Four axioms for coherent risk measures:**

  Translation invariance: **Adding an initial sure amount to the portfolio reduces risk by the same amount.**

  Sub additivity: **"Merger does not create extra risk"**

  Positive Homogeneity: **Doubling all positions doubles the risk.**

  Monotonicity: **Risk is not increased by adding position which has no probability of loss.**

  Artzner et al (1999), "Coherent Measures of Risk", Mathematical Finance, 9(3), 203-228

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Examples of Coherent Measures

- **ES is a coherent risk measure.**
- **The maximum of the expected loss under a set of probability measures or generalized scenarios is a coherent risk measure. (Converse is also true). SPAN is coherent.**
- **VaR is not coherent because it is not subadditive.**

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Axiom of Relevance

- **Artzner et al also proposed:**

  Axiom of Relevance: **Position that can never make a profit but can make a loss has positive risk.**

  Wide Range of scenarios: **Convex hull of generalized scenarios should contain physical and risk neutral probability measures.**

- **In my opinion, SPAN does not satisfy this because of too few scenarios.**

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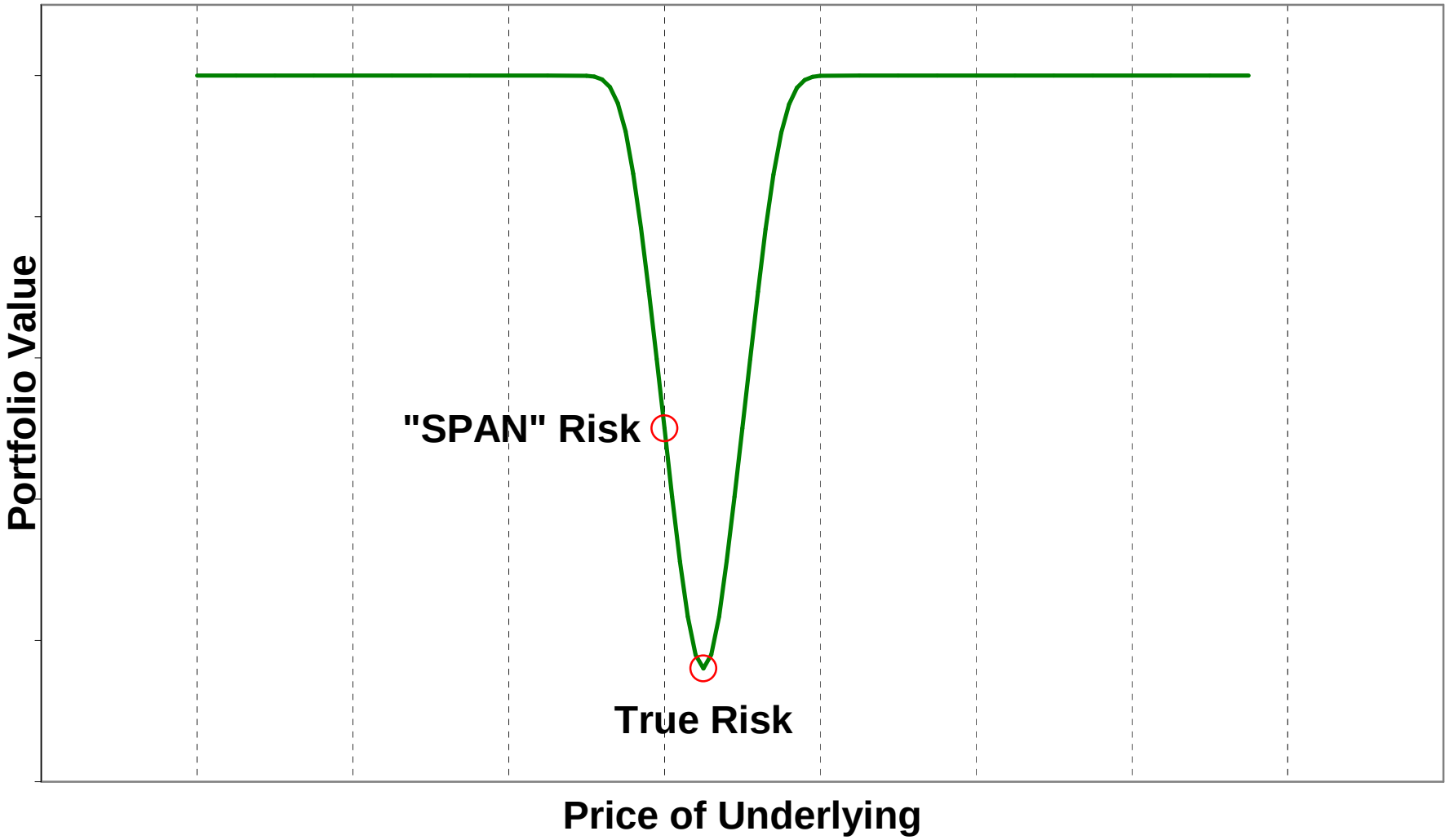
SPAN Underestimates the Risk of a Short Butterfly

Dotted lines are SPAN price scenarios



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Too Few Scenarios in SPAN

- **If price scanning range is set at ±3σ, then there are no scenarios between 0 and σ which covers a probability of 34%.**
- **Possible Solutions:**
  - **Increase number of scenarios (say at each percentile)**
  - **Use a delta-gamma approximation**
- **Probably, we should do both.**

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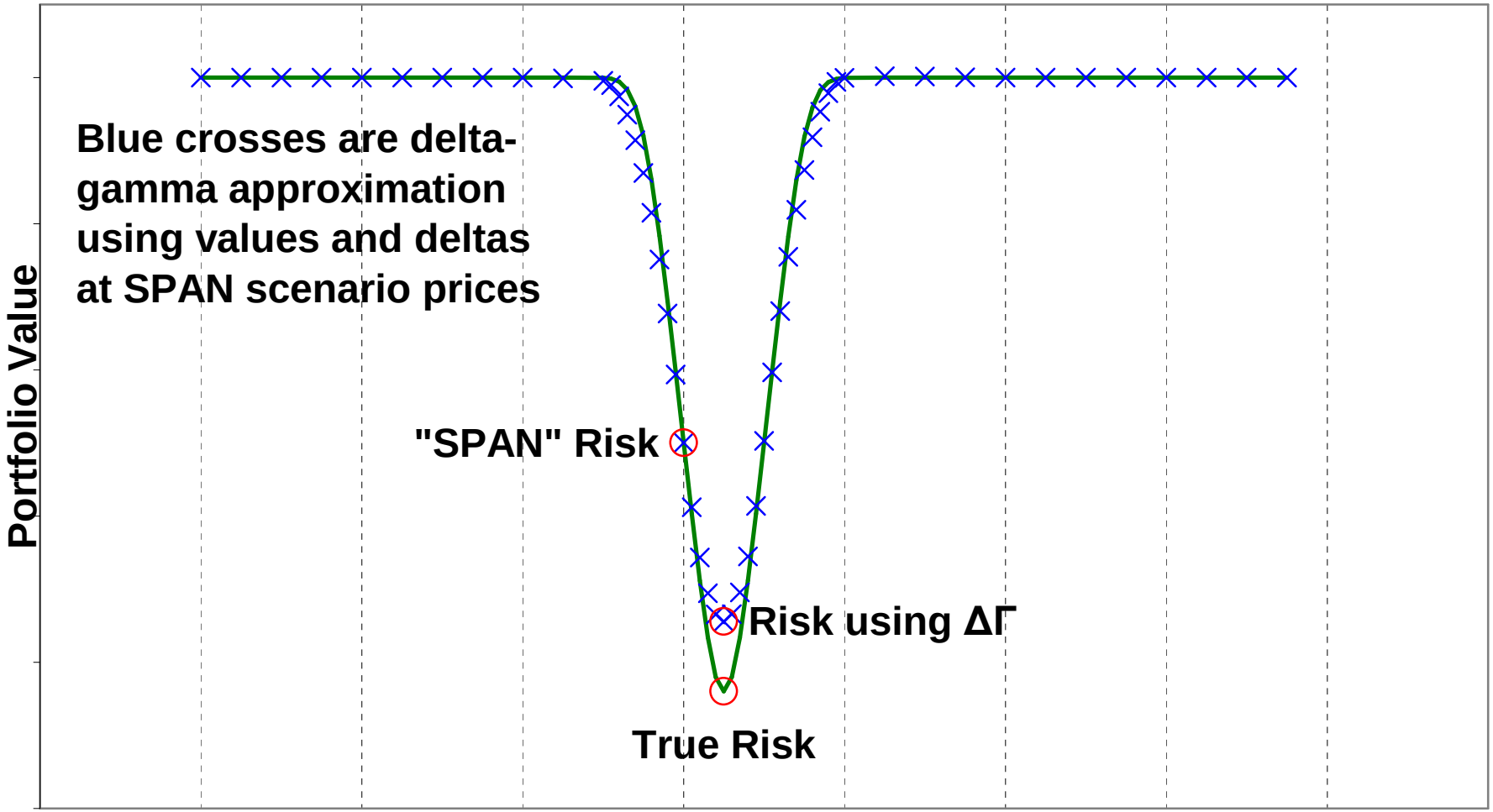
Improved Estimate of the Risk of a Short Butterfly

Dotted lines are SPAN price scenarios



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From VaR to SPAN to ES

- **SPAN is not portfolio VaR, it is more like sum of VaRs eg deep OTM call and put. It is a move towards ES.**
- **Delta-Gamma approximation can be used to compute ES by analytically integrating the polynomial over several sub intervals.**
- **In the tails, ES can be approximated using tail index: h/(h-1) times VaR. Use notional value or delta for aggregation. Indian ETD does this.**
- **All this entails only O(n) complexity.**

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Tail Index

- Normal distribution has exponentially declining tails.
- Fat tails follow power law ~ x^{-h}
- Quasi Maximum Likelihood (QML):
  - Use least squares GARCH estimates
  - Estimate tail index from residuals
  - Consistent estimator + large sample size
- Risk Metrics is a GARCH variant

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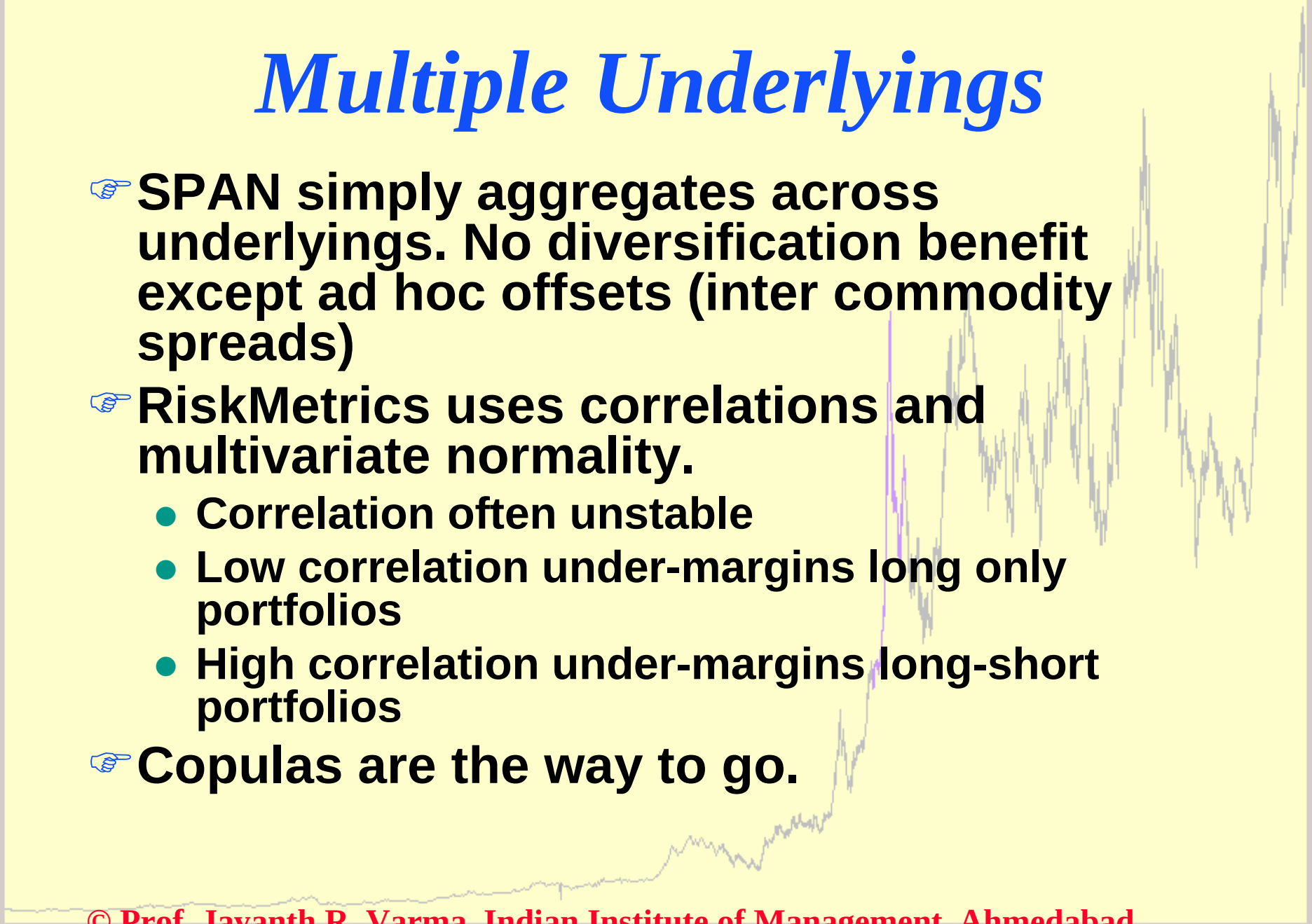
Multiple Underlyings

- **SPAN simply aggregates across underlyings. No diversification benefit except ad hoc offsets (inter commodity spreads)**
- **RiskMetrics uses correlations and multivariate normality.**
  - **Correlation often unstable**
  - **Low correlation under-margins long only portfolios**
  - **High correlation under-margins long-short portfolios**
- **Copulas are the way to go.**



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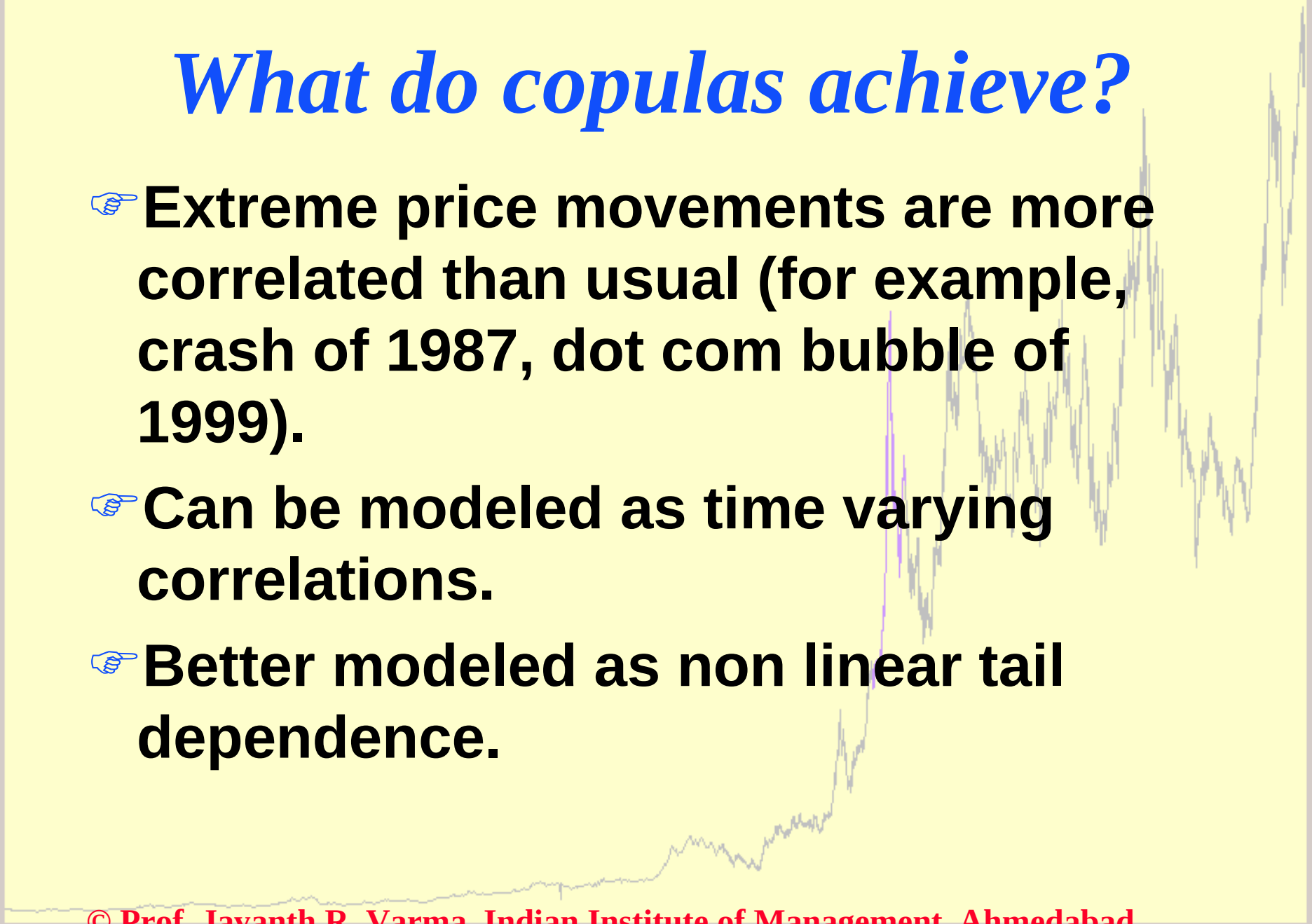
What do copulas achieve?

- **Extreme price movements are more correlated than usual (for example, crash of 1987, dot com bubble of 1999).**
- **Can be modeled as time varying correlations.**
- **Better modeled as non linear tail dependence.**



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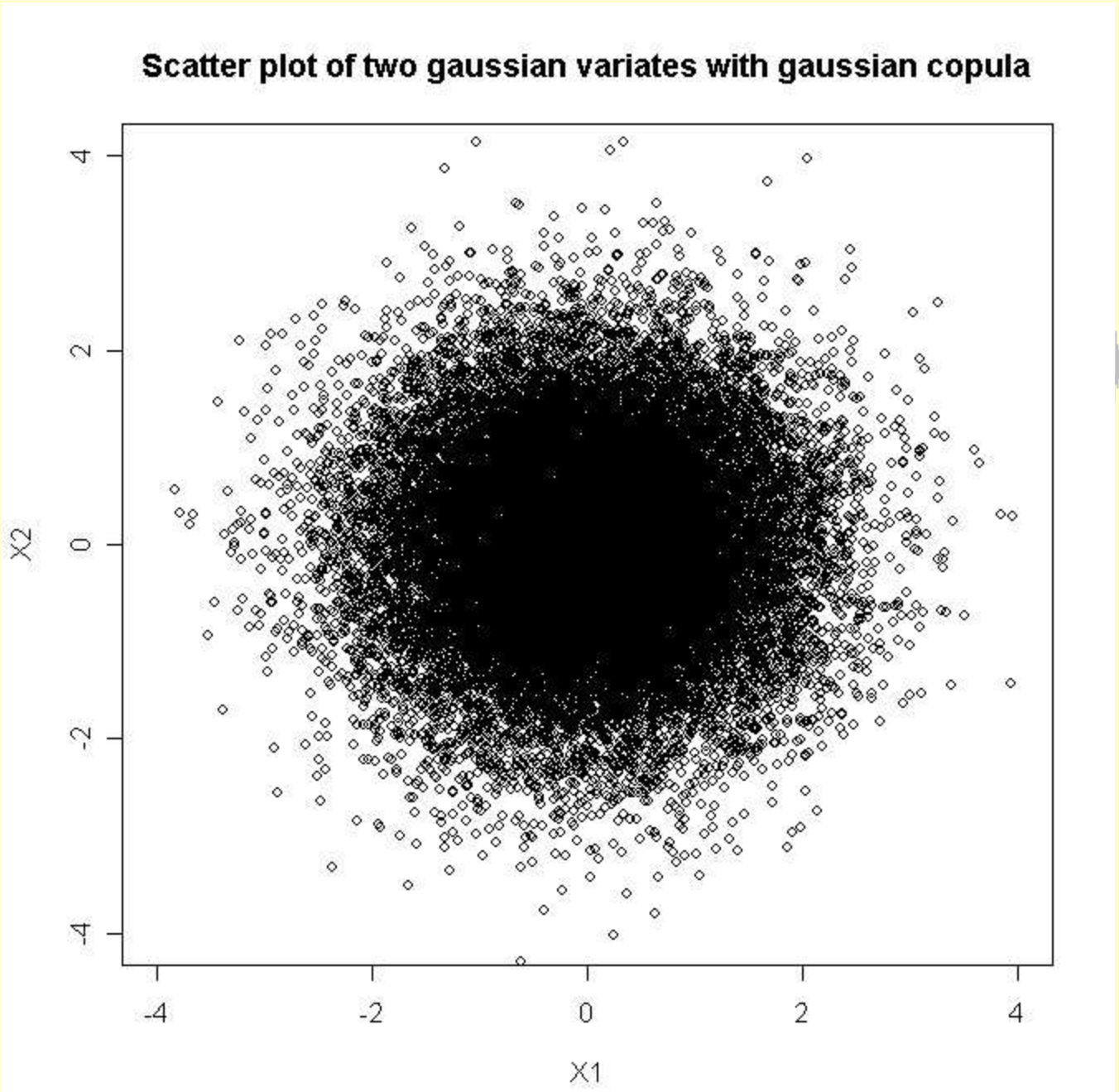

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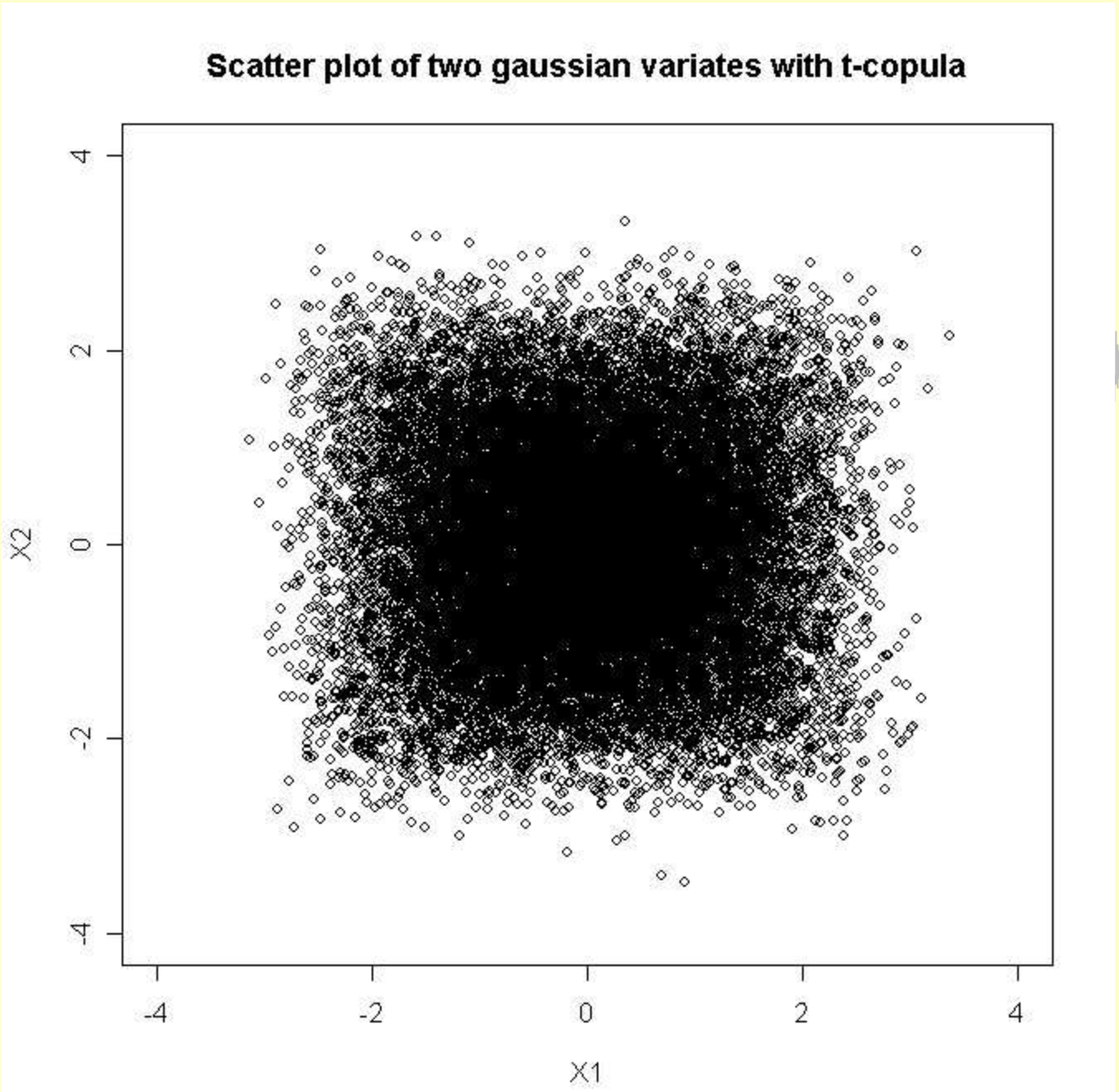

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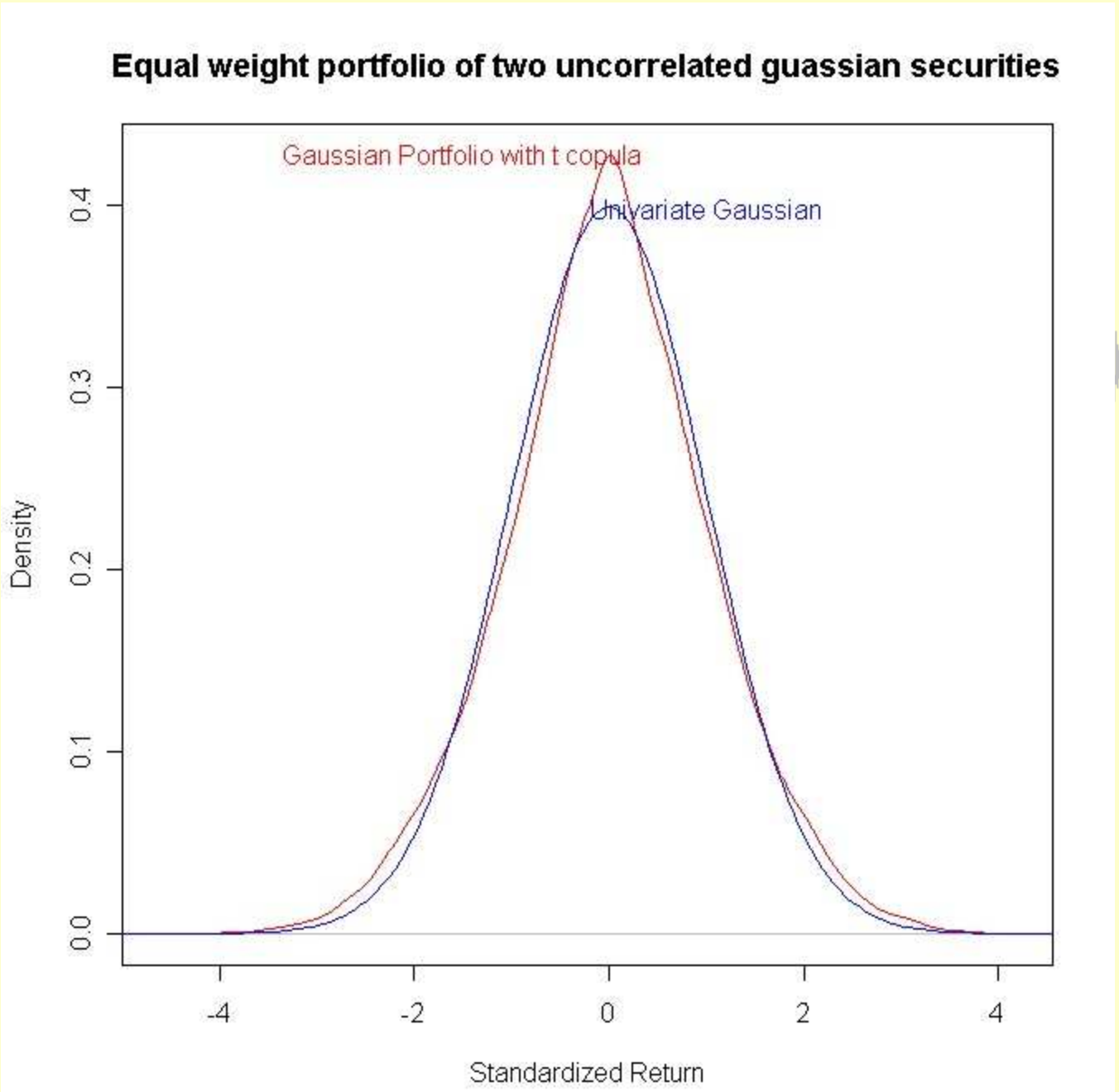

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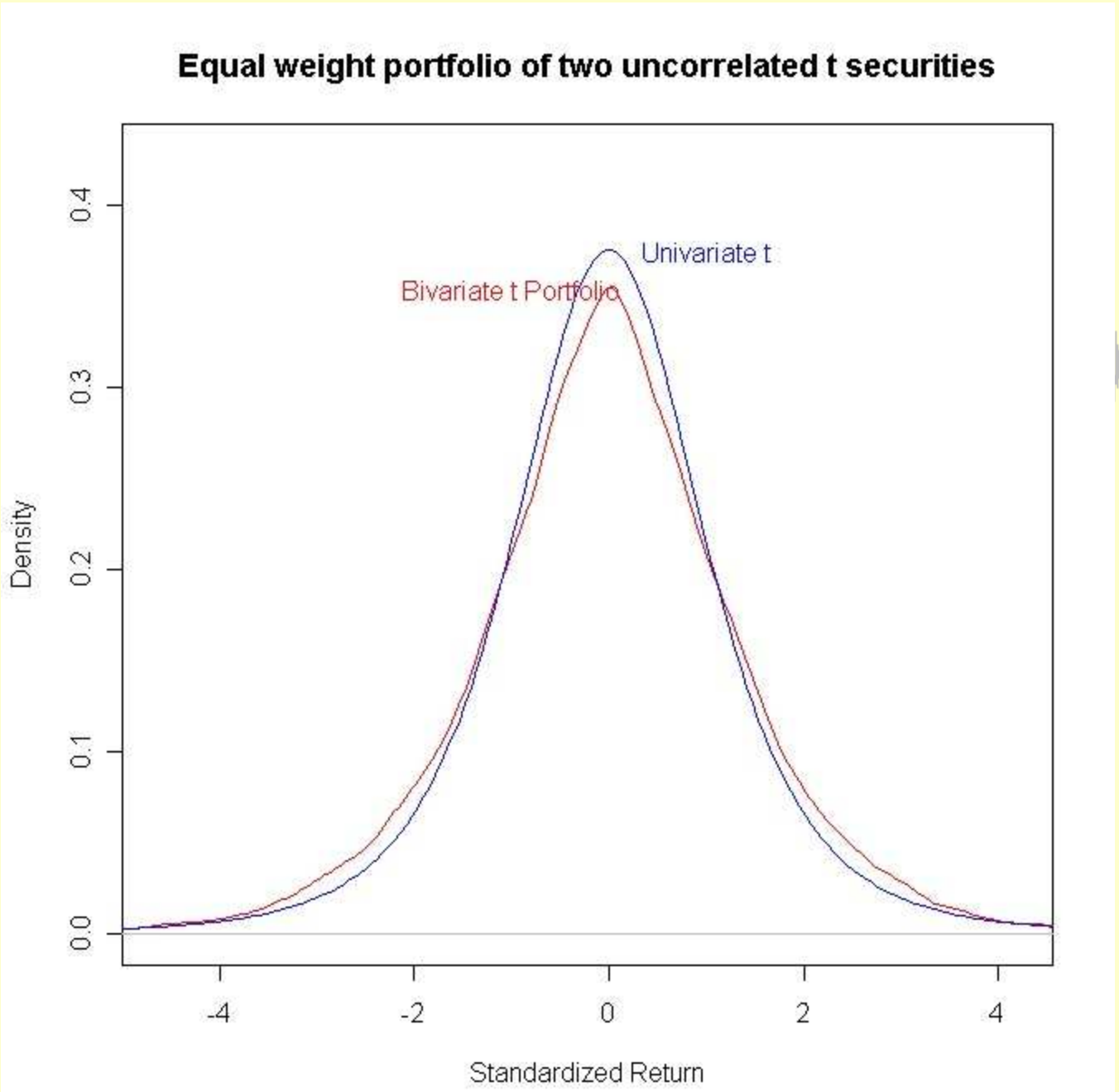

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Choice of copulas

- **Multivariate normality solves curse of dimensionality as portfolio distribution is univariate normal.**
- **Unidimensional mixture of multivariate normals is attractive as it reduces to numerical integral in one dimension.**
- **Multivariate *t* (*t* copula with *t* marginals) is inverse gamma mixture of multivariate normals.**
- **Other mixtures possible. Again the complexity is only O(n) unlike general copulas.**

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Fitting marginal distributions

- To use copulas, we must fit a marginal distribution to the portfolio losses for each underlying and apply copula to these marginals.
- SPAN with enough scenarios approximates the distribution.
- Fit distribution to match the tails well. Match tail quantiles in addition to matching moments.

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Directions for Research

- **Statistical estimation and goodness of fit.**
- **Refinement of algorithms – accuracy and efficiency.**
- **Computational software (open source?)**
- **Advocacy.**

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Another direction – game theory

- If arbitrage is leverage constrained, then arbitrageurs seek under-margined portfolios.
- Two stage game:
  - Exchange moves first – sets margin rules
  - Arbitrageur moves second – chooses portfolios
- Can we solve the game (within O(n) complexity) to set optimal margins?

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Game against nature

- **Systemic risk:**
  - **Exchange is short options on each trader's portfolio with strike equal to portfolio margin.**
  - **What price scenarios create worst loss to exchange (aggregated across all traders)?**
  - **Add these scenarios to margining system dynamically**
- **Three stage game:**
  - **Traders choose portfolios**
  - **Exchange decides on "special" margins or "special" margining scenarios**
  - **Nature (market?) reveals new prices**
- **Can we solve this game within O(n) complexity?**

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