Standard Test Problems for Single-Objective Bilevel Optimization

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1 Introduction

This file contains a description of a set of 10 standard bilevel test problems chosen from the literature [7, 1, 2, 3, 6, 5, 4, 10]. Most of these problems are constrained problems with relatively smaller number of variables. The problems are numbered as TP1 to TP10 in [9], where they have been extensively used along with SMD[8] test-suite for comparing different algorithms. The codes for these problems may be accessed from the website `http://bilevel.org`.

2 Standard test problems

Tables 1 and 2 define a set of 10 standard test problems. The dimensions of the upper (n) and lower (m) level variables are given in the first column, and the problem formulation is defined in the second column. The third column provides the best known solution available in the literature for the chosen test problems. Please note that the upper level variable is denoted as x, and the lower level variable is denoted as y.

Table 1: Description of the selected standard test problems (TP1-TP5).

Problem	Formulation	Best Known Sol.
TP1 $n=2\ m=2$	$\underset{(x,y)}{\text{Minimize}}\ F(x,y)=(x_1-30)^2+(x_2-20)^2-20y_1+20y_2,$ s.t. $y\in\underset{(y)}{\text{argmin}}\left\{\begin{array}{l} f(x,y)=(x_1-y_1)^2+(x_2-y_2)^2 \\ 0\le y_i\le 10,\quad i=1,2\end{array}\right\},$ $x_1+2x_2\ge 30, x_1+x_2\le 25, x_2\le 15$	$F=225.0$ $f=100.0$
TP2 $n=2\ m=2$	$\underset{(x,y)}{\text{Minimize}}\ F(x,y)=2x_1+2x_2-3y_1-3y_2-60,$ s.t. $y\in\underset{(y)}{\text{argmin}}\left\{\begin{array}{l} f(x,y)=(y_1-x_1+20)^2+(y_2-x_2+20)^2 \\ x_1-2y_1\ge 10, x_2-2y_2\ge 10 \\ -10\ge y_i\ge 20,\quad i=1,2\end{array}\right\},$ $x_1+x_2+y_1-2y_2\le 40,$ $0\le x_i\le 50,\quad i=1,2.$	$F=0.0$ $f=100.0$
TP3 $n=2\ m=2$	$\underset{(x,y)}{\text{Minimize}}\ F(x,y)=-(x_1)^2-3(x_2)^2-4y_1+(y_2)^2,$ s.t. $y\in\underset{(y)}{\text{argmin}}\left\{\begin{array}{l} f(x,y)=2(x_1)^2+(y_1)^2-5y_2 \\ (x_1)^2-2x_1+(x_2)^2-2y_1+y_2\ge -3 \\ x_2+3y_1-4y_2\ge 4 \\ 0\le y_i,\quad i=1,2\end{array}\right\},$ $(x_1)^2+2x_2\le 4,$ $0\le x_i,\quad i=1,2$	$F=-18.6787$ $f=-1.0156$
TP4 $n=2\ m=3$	$\underset{(x,y)}{\text{Minimize}}\ F(x,y)=-8x_1-4x_2+4y_1-40y_2-4y_3,$ s.t. $y\in\underset{(y)}{\text{argmin}}\left\{\begin{array}{l} f(x,y)=x_1+2x_2+y_1+y_2+2y_3 \\ y_2+y_3-y_1\le 1 \\ 2x_1-y_1+2y_2-0.5y_3\le 1 \\ 2x_2+2y_1-y_2-0.5y_3\le 1 \\ 0\le y_i,\quad i=1,2,3\end{array}\right\},$ $0\le x_i,\quad i=1,2$	$F=-29.2$ $f=3.2$
TP5 $n=2\ m=2$	$\underset{(x,y)}{\text{Minimize}}\ F(x,y)=rt(x)x-3y_1-4y_2+0.5t(y)y,$ s.t. $y\in\underset{(y)}{\text{argmin}}\left\{\begin{array}{l} f(x,y)=0.5t(y)hy-t(b(x))y \\ -0.333y_1+y_2-2\le 0 \\ y_1-0.333y_2-2\le 0 \\ 0\le y_i,\quad i=1,2\end{array}\right\},$ where $h=\begin{pmatrix}1 & 3\\ 3 & 10\end{pmatrix}, b(x)=\begin{pmatrix}-1 & 2\\ 3 & -3\end{pmatrix}x, r=0.1$ $t(\cdot)$ denotes transpose of a vector	$F=-3.6$ $f=-2.0$

Table 2: Description of the selected standard test problems (TP6-TP10).

Problem	Formulation	Best Known Sol.
TP6 $n = 1\ m = 2$	$\underset{(x,y)}{\text{Minimize}}\ F(x,y) = (x_1 - 1)^2 + 2y_1 - 2x_1,$ s.t. $y \in \underset{(y)}{\text{argmin}} \left\{ \begin{array}{l} f(x,y) = (2y_1 - 4)^2 + \\ (2y_2 - 1)^2 + x_1 y_1 \\ 4x_1 + 5y_1 + 4y_2 \leq 12 \\ 4y_2 - 4x_1 - 5y_1 \leq -4 \\ 4x_1 - 4y_1 + 5y_2 \leq 4 \\ 4y_1 - 4x_1 + 5y_2 \leq 4 \\ 0 \leq y_i, \quad i = 1,2 \end{array} \right\},$ $0 \leq x_1$	$F = -1.2091$ $f = 7.6145$
TP7 $n = 2\ m = 2$	$\underset{(x,y)}{\text{Minimize}}\ F(x,y) = -\frac{(x_1+y_1)(x_2+y_2)}{1+x_1y_1+x_2y_2},$ s.t. $y \in \underset{(y)}{\text{argmin}} \left\{ \begin{array}{l} f(x,y) = \frac{(x_1+y_1)(x_2+y_2)}{1+x_1y_1+x_2y_2} \\ 0 \leq y_i \leq x_i, \quad i = 1,2 \end{array} \right\},$ $(x_1)^2 + (x_2)^2 \leq 100$ $x_1 - x_2 \leq 0$ $0 \leq x_i, \quad i = 1,2$	$F = -1.96$ $f = 1.96$
TP8 $n = 2\ m = 2$	$\underset{(x,y)}{\text{Minimize}}\ F(x,y) = \lvert 2x_1 + 2x_2 - 3y_1 - 3y_2 - 60 \rvert,$ s.t. $y \in \underset{(y)}{\text{argmin}} \left\{ \begin{array}{l} f(x,y) = (y_1 - x_1 + 20)^2 + \\ (y_2 - x_2 + 20)^2 \\ 2y_1 - x_1 + 10 \leq 0 \\ 2y_2 - x_2 + 10 \leq 0 \\ -10 \leq y_i \leq 20, \quad i = 1,2 \end{array} \right\},$ $x_1 + x_2 + y_1 - 2y_2 \leq 40$ $0 \leq x_i \leq 50, \quad i = 1,2$	$F = 0.0$ $f = 100.0$
TP9 $n = 10\ m = 10$	$\underset{(x,y)}{\text{Minimize}}\ F(x,y) = \sum_{i=1}^{10} \left(\lvert x_i - 1 \rvert + \lvert y_i \rvert \right),$ s.t. $y \in \underset{(y)}{\text{argmin}} \left\{ \begin{array}{l} f(x,y) = e^{\left(1 + \frac{1}{4000} \sum_{i=1}^{10} (y_i)^2 - \prod_{i=1}^{10} \cos(\frac{y_i}{\sqrt{i}})\right) \sum_{i=1}^{10} (x_i)^2} \\ \quad -\pi \leq y_i \leq \pi, \quad i = 1,2\ldots,10 \end{array} \right\},$	$F = 0.0$ $f = 1.0$
TP10 $n = 10\ m = 10$	$\underset{(x,y)}{\text{Minimize}}\ F(x,y) = \sum_{i=1}^{10} \left(\lvert x_i - 1 \rvert + \lvert y_i \rvert \right),$ s.t. $y \in \underset{(y)}{\text{argmin}} \left\{ \begin{array}{l} f(x,y) = e^{\left(1 + \frac{1}{4000} \sum_{i=1}^{10} (y_i x_i)^2 - \prod_{i=1}^{10} \cos(\frac{y_i x_i}{\sqrt{i}})\right)} \\ \quad -\pi \leq y_i \leq \pi, \quad i = 1,2\ldots,10 \end{array} \right\},$	$F = 0.0$ $f = 1.0$

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