

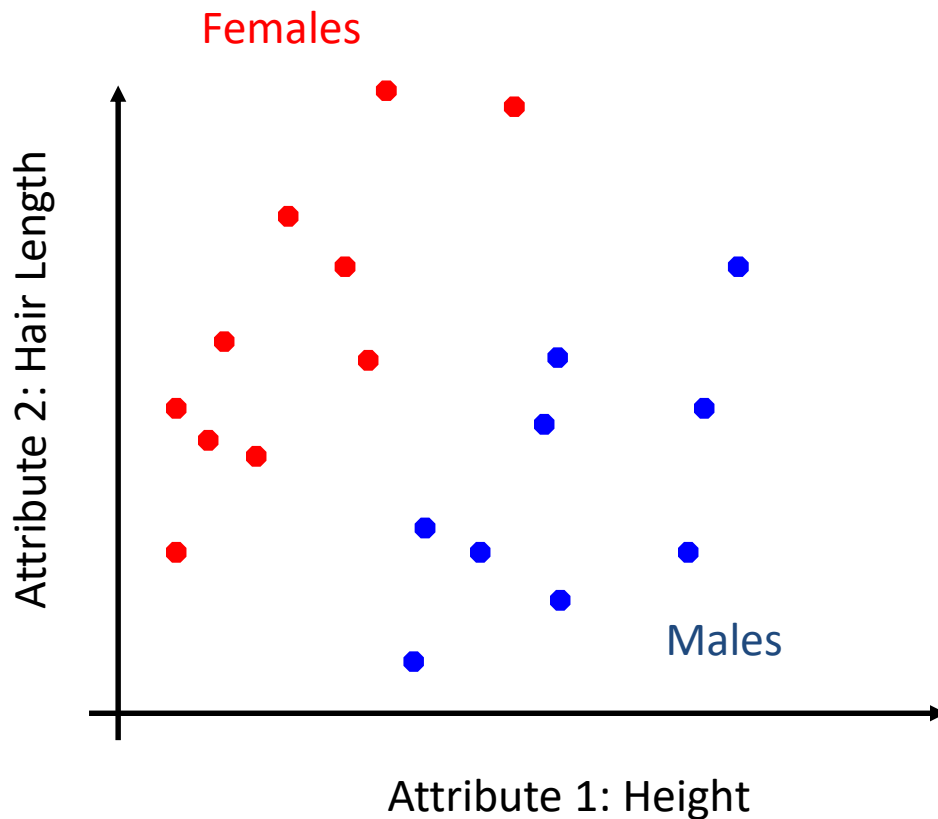
Support Vector Machines

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Support Vector Machines (SVM)



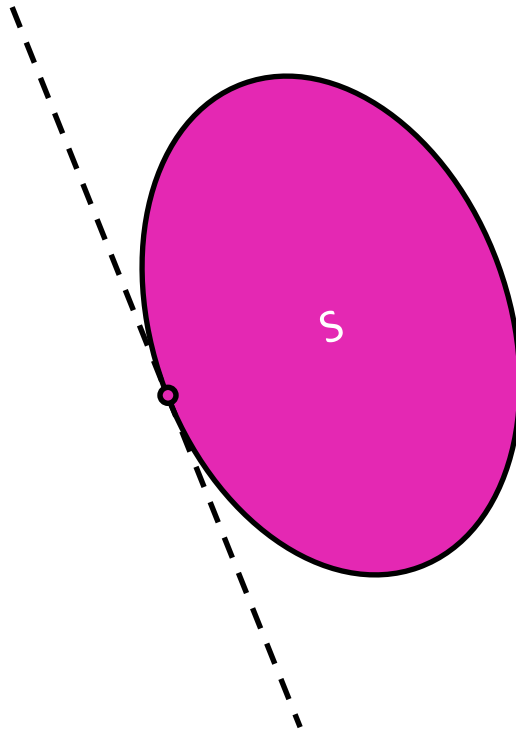
I have some objects with two attributes

It could be people with their heights and hair length

There are some points where you know the males and females

You want to train a robot based on these available points to classify people based on these attributes

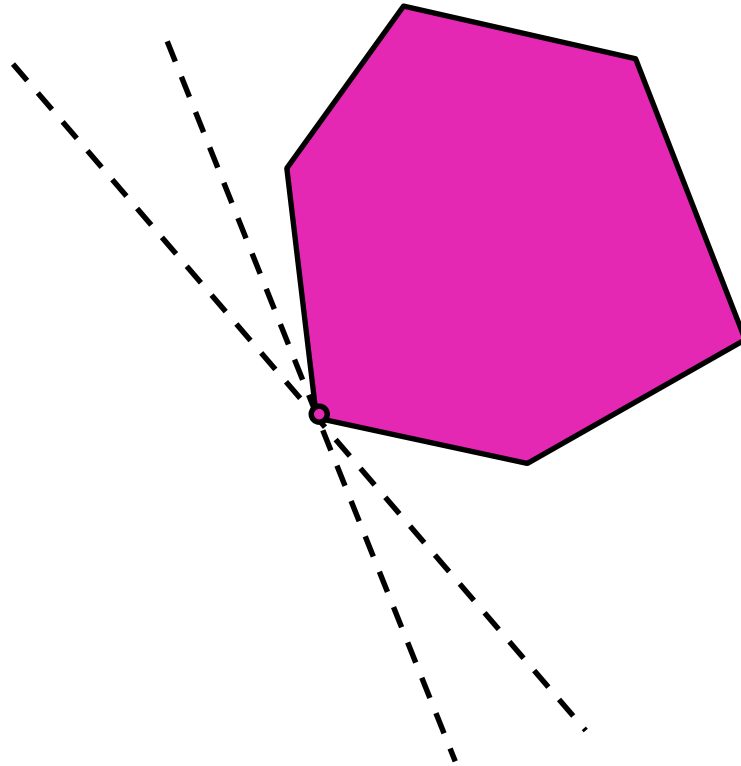
Supporting Hyperplane



A supporting hyperplane of a set S that has both of the following two properties:

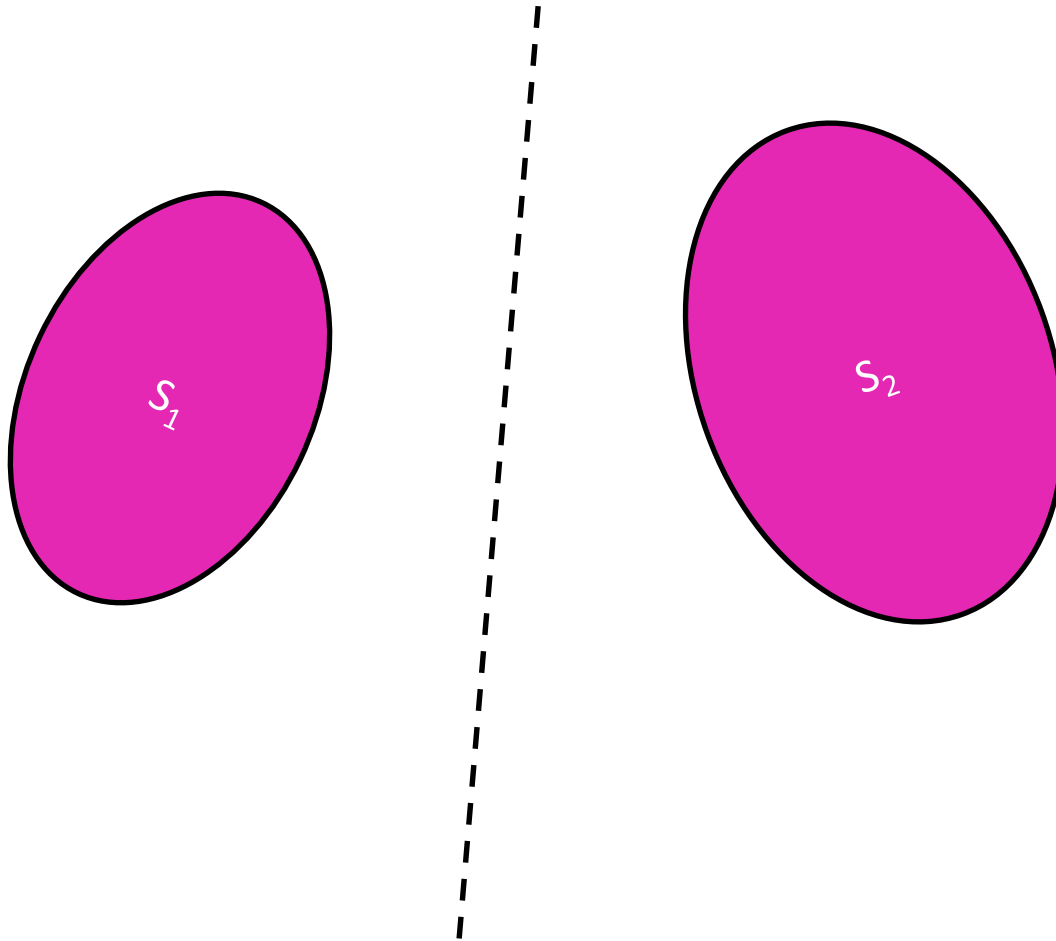
- S is entirely contained in one of the half-spaces
- S has at least one boundary-point on the hyperplane

Supporting Hyperplane

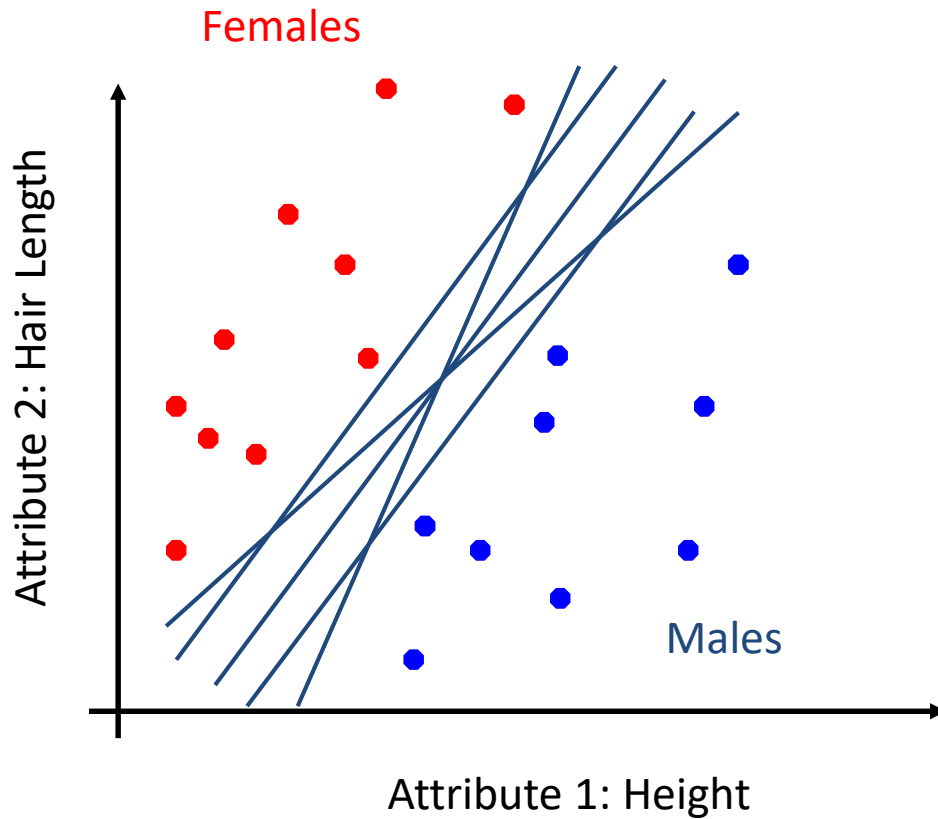


More than one hyperplane may be possible that meets the supporting hyperplane criteria

Separating Hyperplane



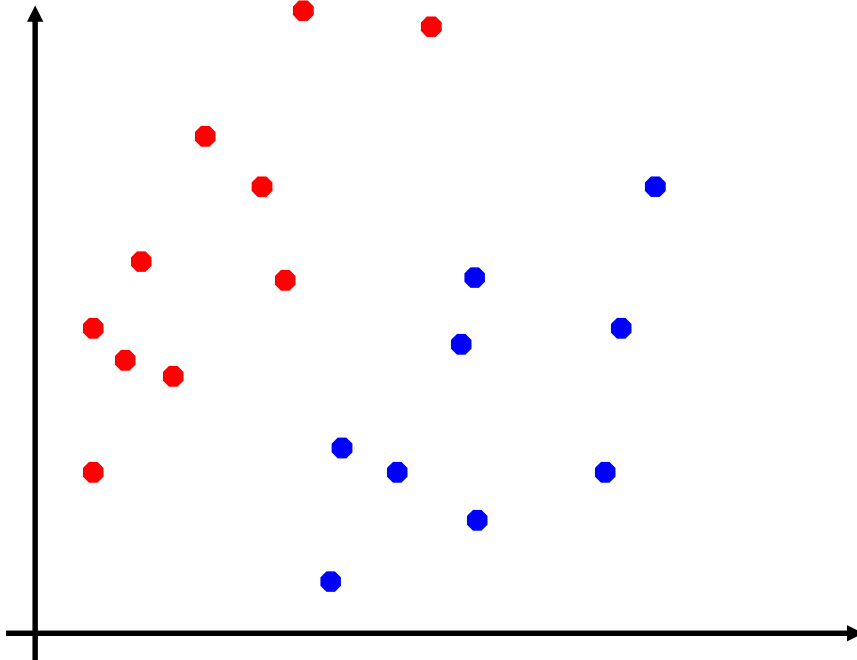
Support Vector Machines (SVM)



Multiple planes are possible

Which one to choose?

Support Vector Machines (SVM)

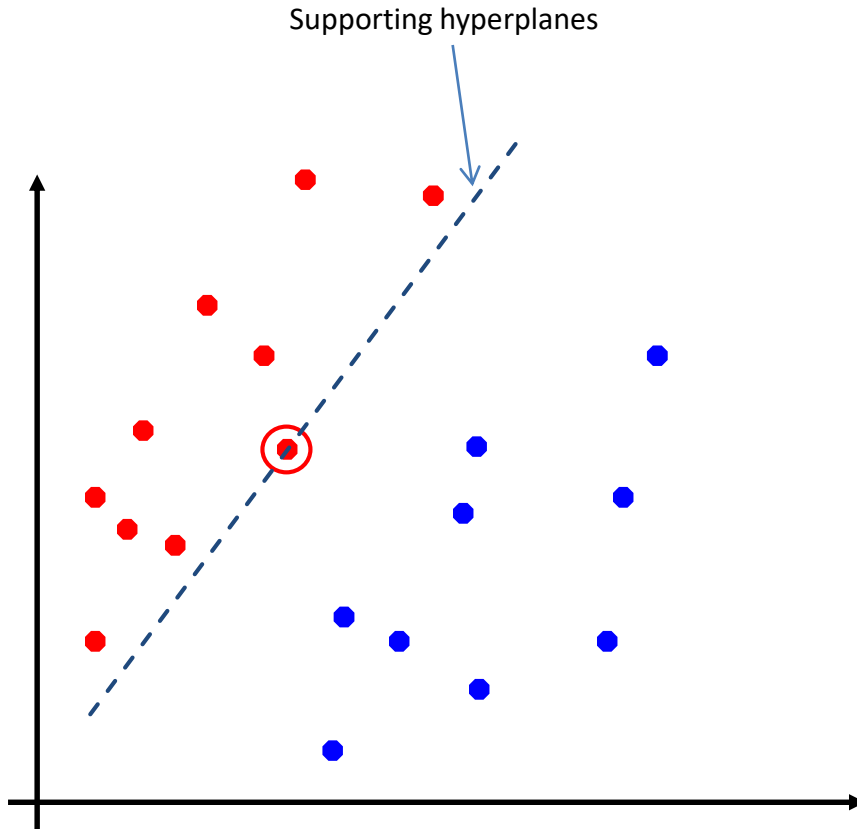


Choose the maximum margin hyperplane: Idea behind SVM

A maximum margin hyperplane separates the data points maximally

SVM tries to find the maximally separating plane

Support Vector Machines (SVM)

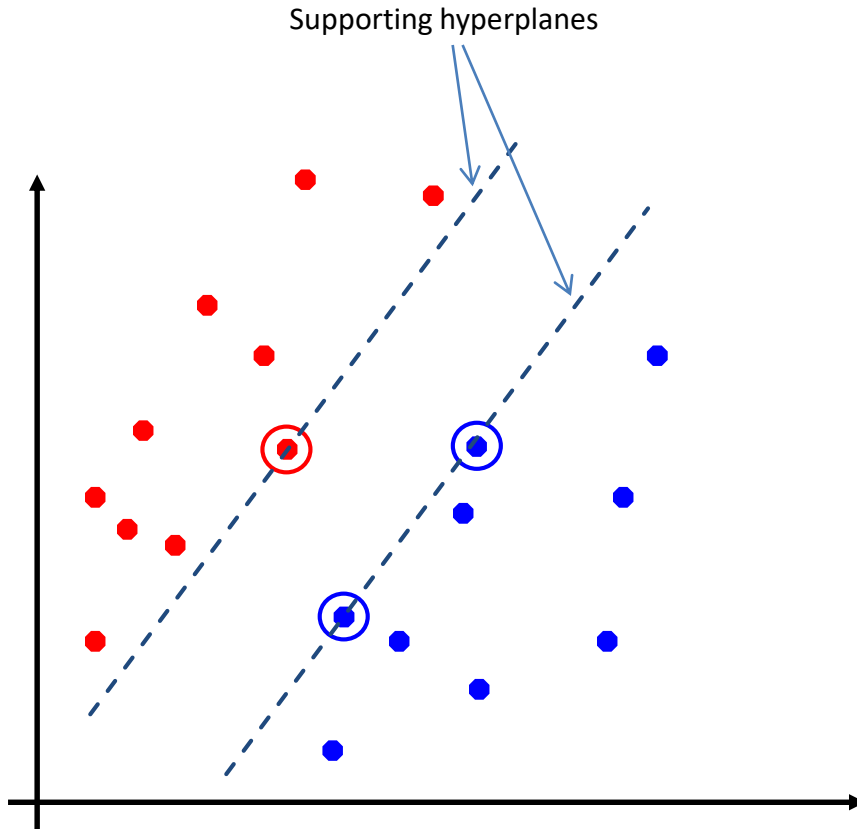


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Support Vector Machines (SVM)

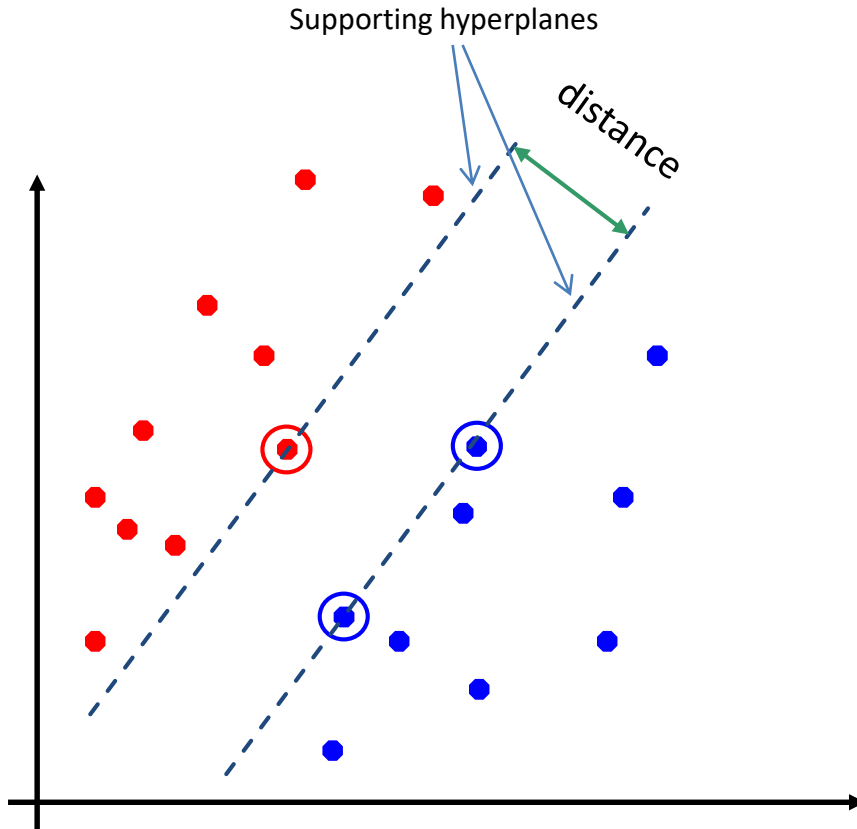


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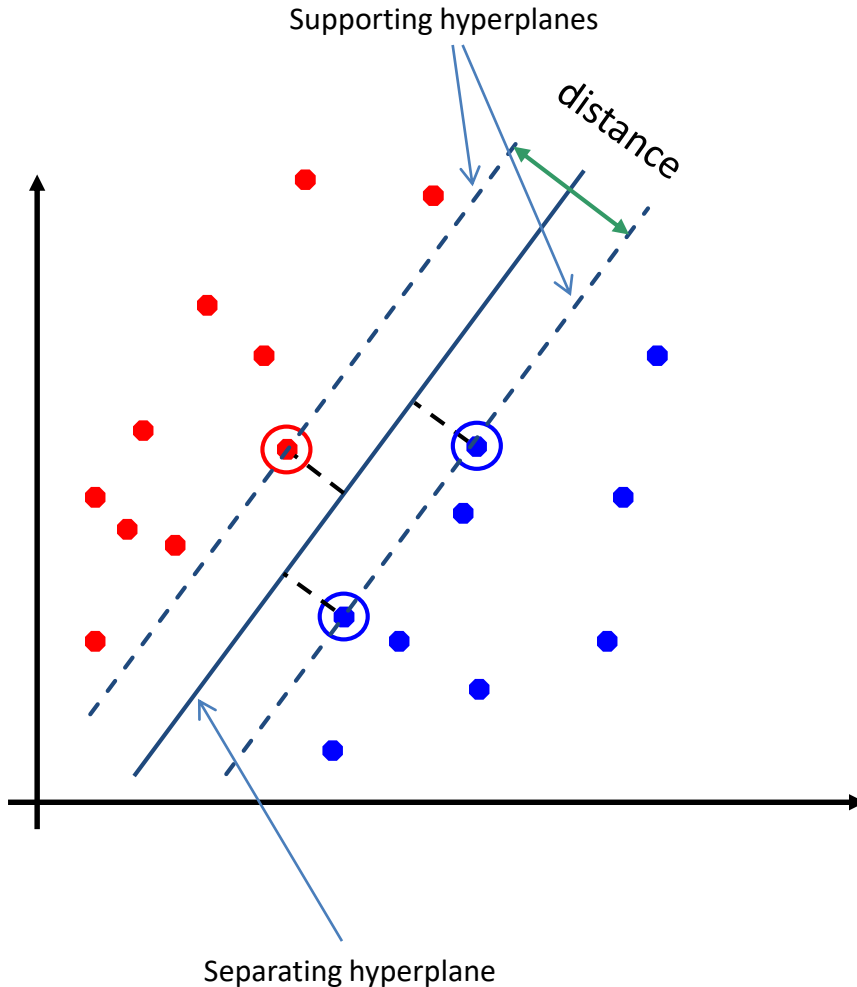


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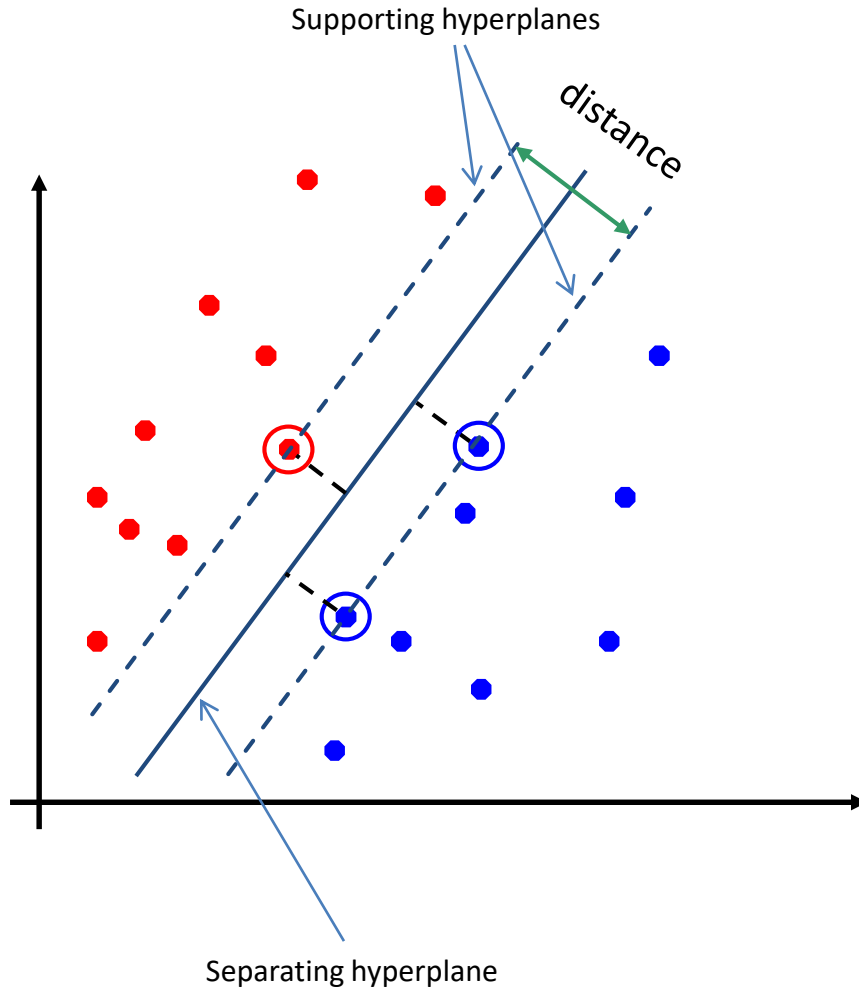


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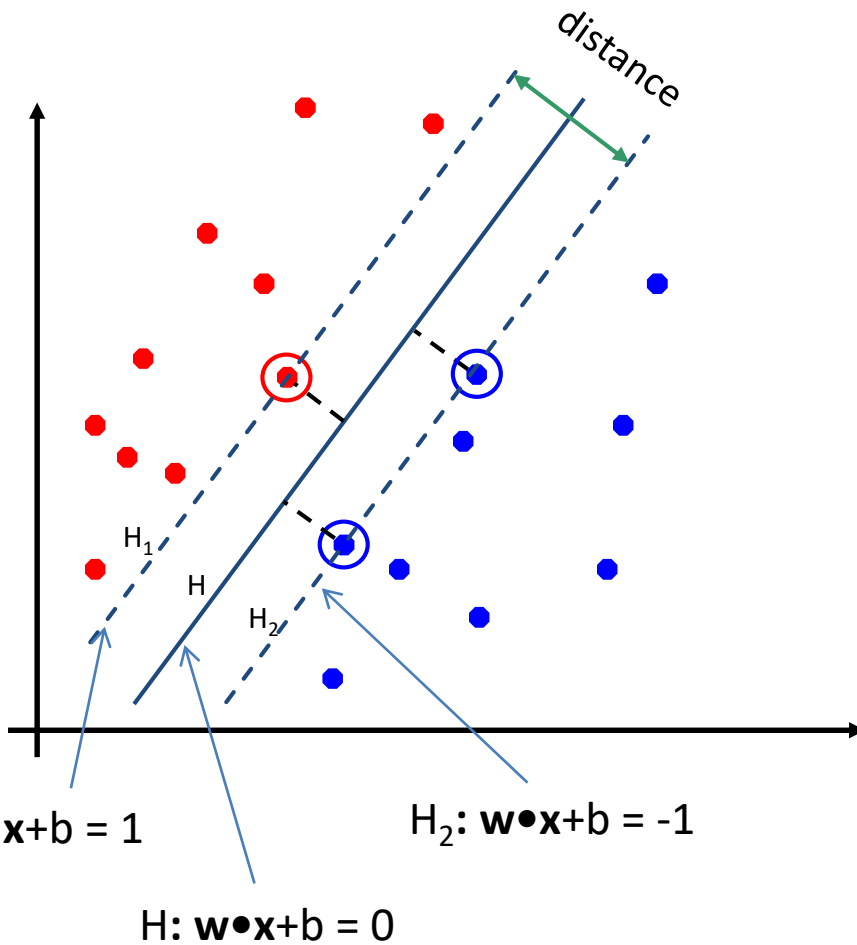


Choose the maximum margin hyperplane: Idea behind SVM

A maximum margin hyperplane separates the data points maximally

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Support Vector Machines (SVM)



The distance from a point $\mathbf{x}_0 = (x_0, y_0)$ to a line: $Ax + By + c = 0$ is given as:
 $|A x_0 + B y_0 + c| / \sqrt{A^2 + B^2}$

The distance between H and H_1 is:
 $1 / \|\mathbf{w}\|$

The distance between H_1 and H_2 is:
 $2 / \|\mathbf{w}\|$

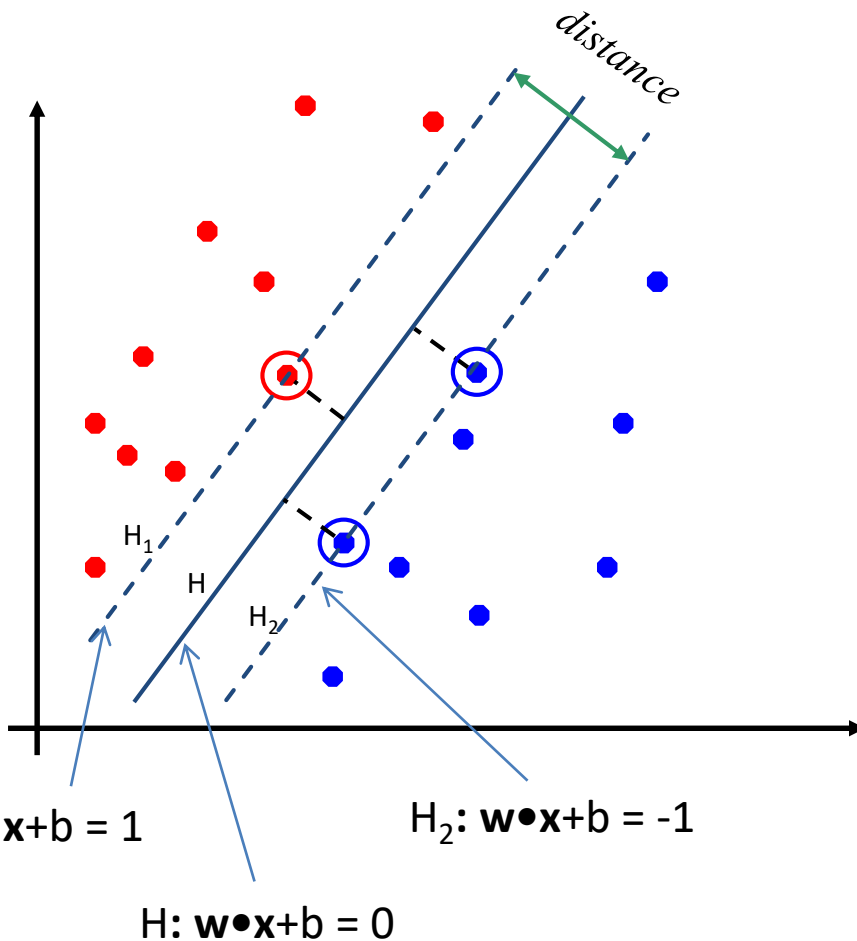
Maximizing the margin is same as minimizing $\|\mathbf{w}\|$ with the following condition:

$\mathbf{x} \bullet \mathbf{w} + b \geq +1$ when point lies in blue class

$\mathbf{x} \bullet \mathbf{w} + b \leq -1$ when point lies in red class

An optimization is performed to find appropriate values of w and b .

Support Vector Machines (SVM)

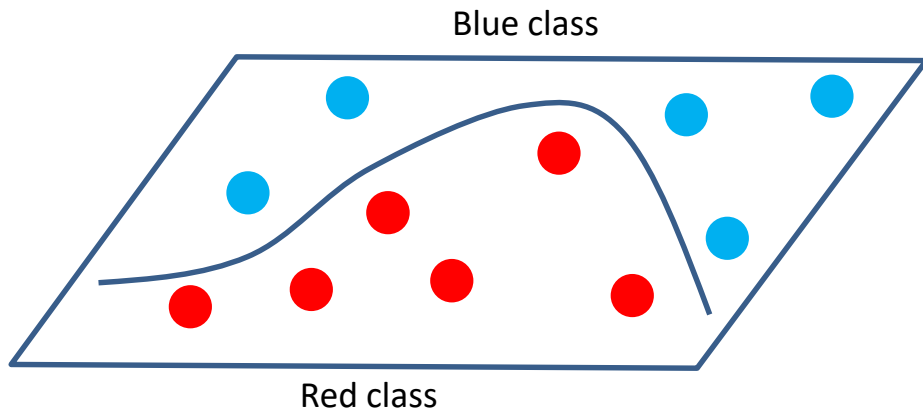


A more concise notation is as follows:

Find \mathbf{w} and b such that
 $\frac{1}{2} \mathbf{w}^T \mathbf{w}$ is minimized;
and for all $\{(\mathbf{x}_i, y_i)\}$: $y_i (\mathbf{w}^T \mathbf{x}_i + b) \geq 1$

Where \mathbf{x}_i represents an object, and
 y_i represents its class

How about the following case?

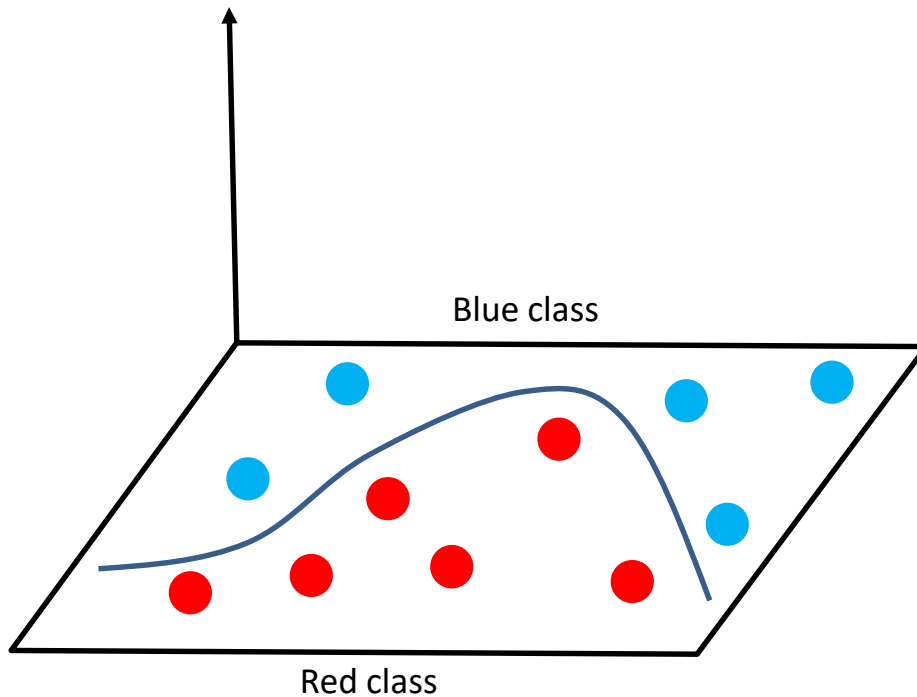


A linear model cannot be fitted

However, SVM is only capable of fitting hyperplanes

Solution: Take the dataset into higher dimension by adding a third dimension in the data

How about the following case?



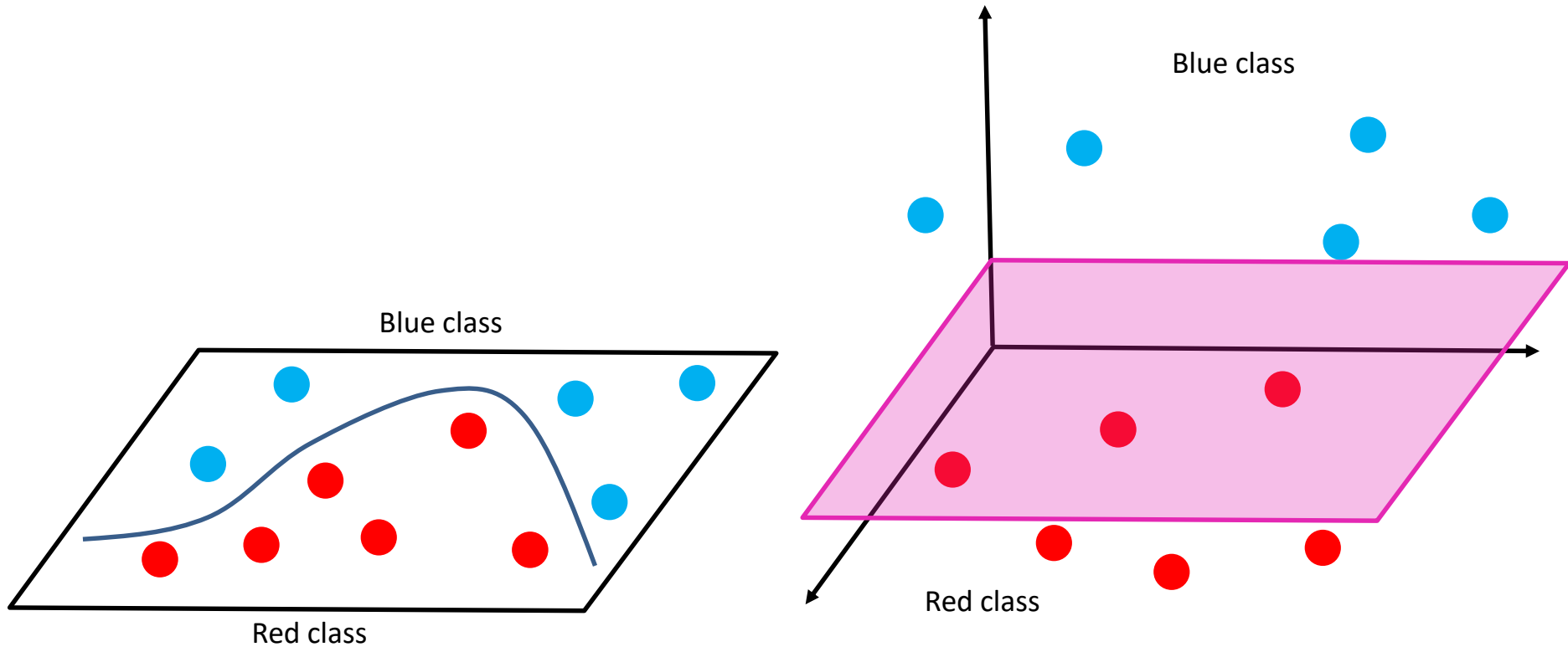
A linear model cannot be fitted

However, SVM is only capable of fitting hyperplanes

Solution: Take the dataset into higher dimension by adding a third dimension in the data

Third dimension is a function of the first two dimensions

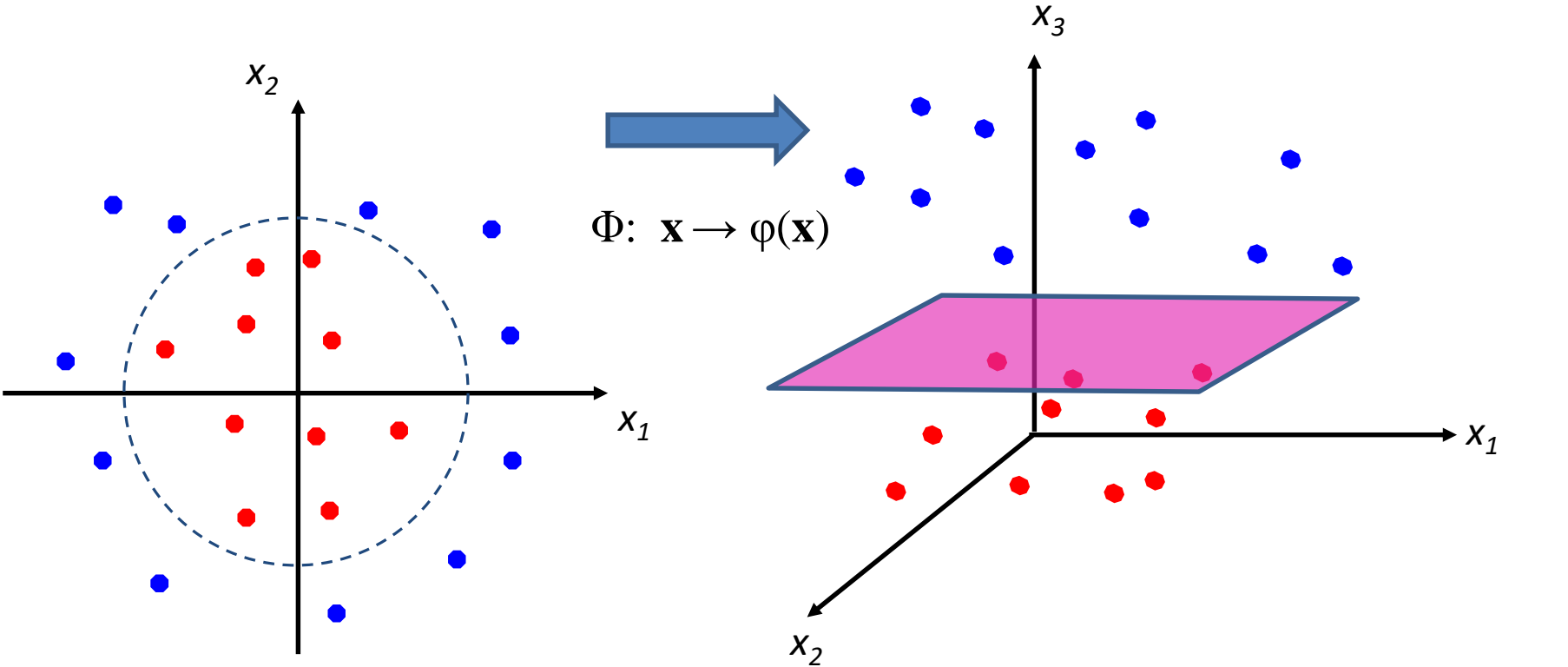
Non-linear SVM



A kernel function is used to transform lower dimensional data to higher dimensional data

A hyperplane can be added now.

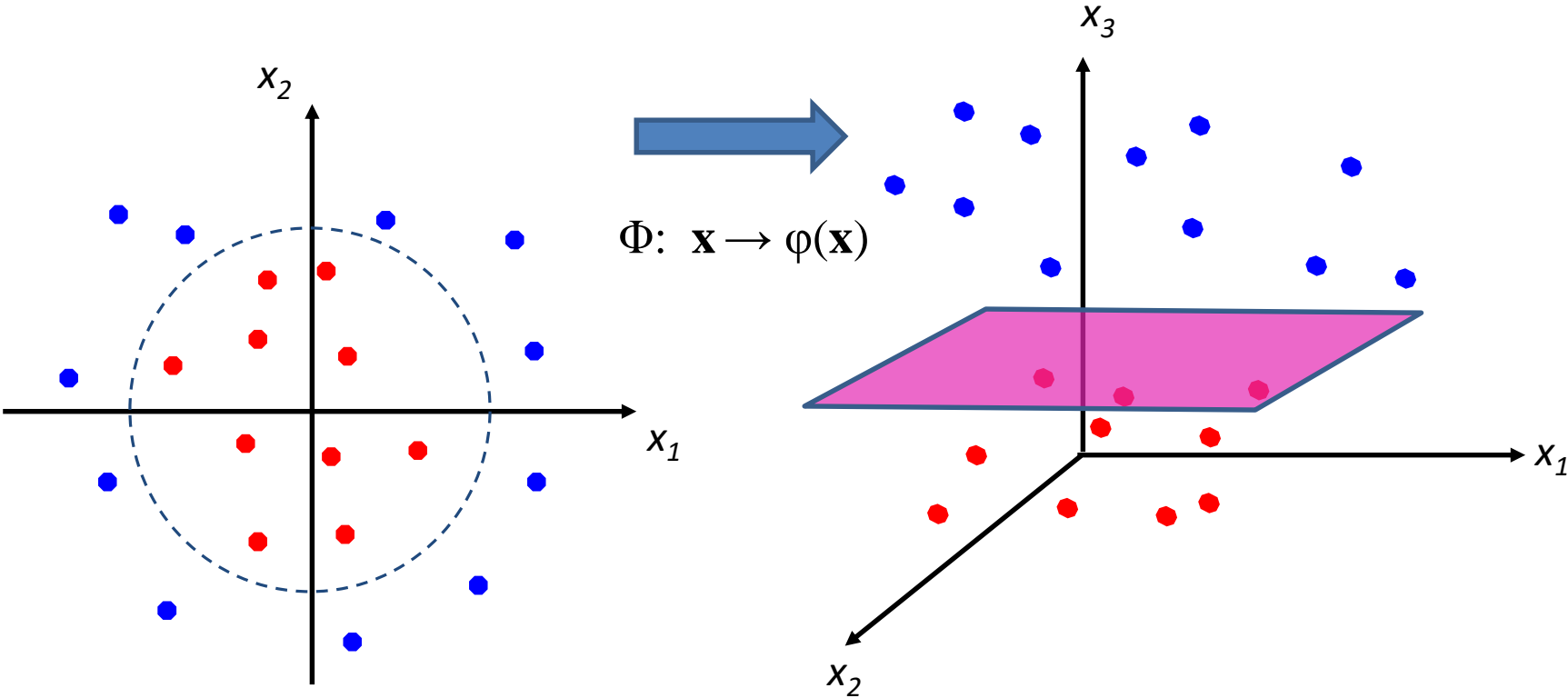
Non-linear SVM



This is the third dimension

$$\text{Kernel } \phi((x_1, x_2)) = (x_1, x_2, x_1^2 + x_2^2)$$

Non-linear SVM



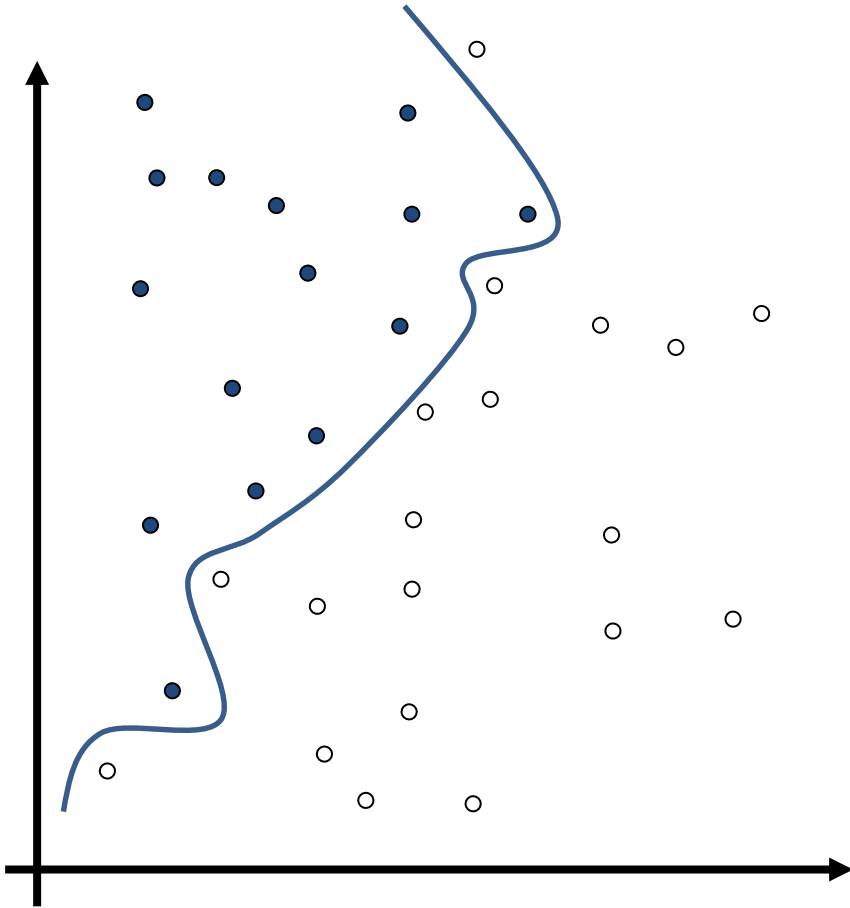
Mapping function $\phi((x_1, x_2)) = (x_1, x_2, x_1^2 + x_2^2)$

The separating hyperplane in higher dimensions is actually a circle in the lower dimension in this case

Kernels

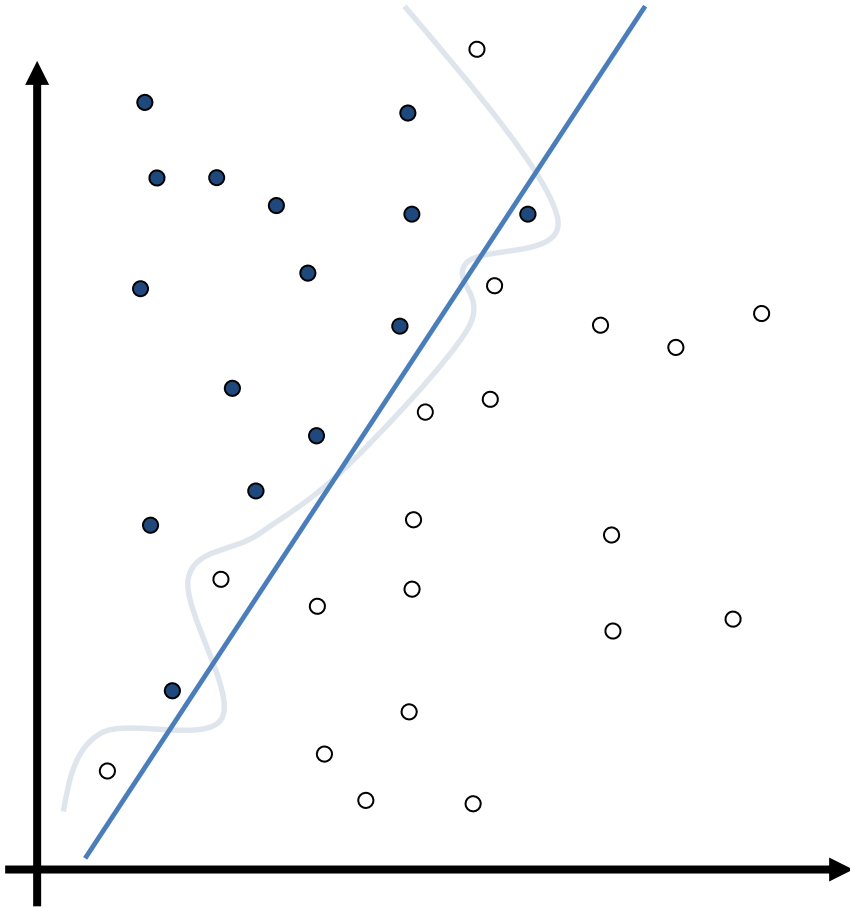
- Kernels can be of different types, for example:
 - Polynomial kernel: $K(w,x) = (w \bullet x + 1)^p$, where p is a parameter
 - Gaussian kernel: $K(w,x) = \exp(-\|w-x\|^2/2\sigma^2)$ where σ is a parameter

Simple vs Complex Kernels



- Non-linear Kernel: All data points are classified correctly
 - No training error
- Is it a good idea in case of this example?

Simple vs Complex Kernels



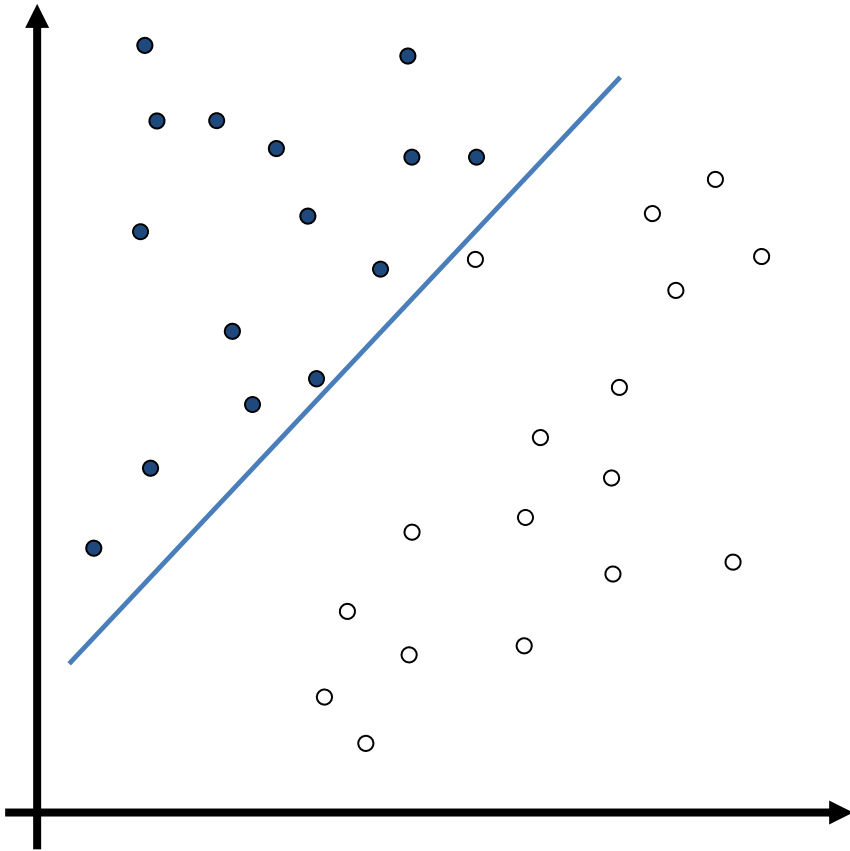
- Non-linear Kernel: All data points are classified correctly
 - No training error
- Is it a good idea in case of this example?
- The straight line serves the purpose of dividing the two classes
- Non-linear Kernels may lead to:

Overfitting!

Soft Margin vs Hard Margin

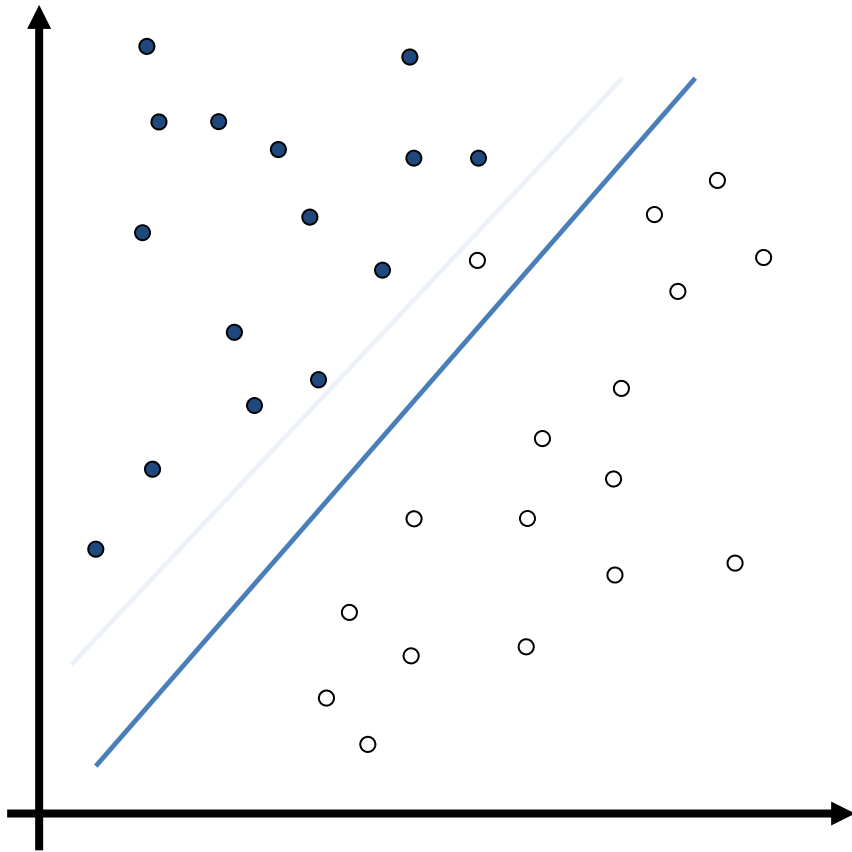
- Hard margin classification means that you want to place the hyperplane such that there is complete classification without any errors
- Soft margin classification means that you allow some errors by not caring if some of the points are misclassified

Soft Margin vs Hard Margin



- Hard Margin: All data points are classified correctly
 - No training error
- Is it a good idea in case of this example?

Soft Margin vs Hard Margin



- Hard Margin: All data points are classified correctly
 - No training error
- Is it a good idea in case of this example?
- A soft margin plane may generalize better than the hard margin plane
- A hard margin plane may lead to:

Overfitting!

Soft Margin

- Locate the hyperplane such that it not only maximally separates the two classes but also tries to minimize the errors e_i

Original Formulation:

Find \mathbf{w} and b such that

$\frac{1}{2} \mathbf{w}^T \mathbf{w}$ is minimized and for all $\{(\mathbf{x}_i, y_i)\}$

$$y_i (\mathbf{w}^T \mathbf{x}_i + b) \geq 1$$

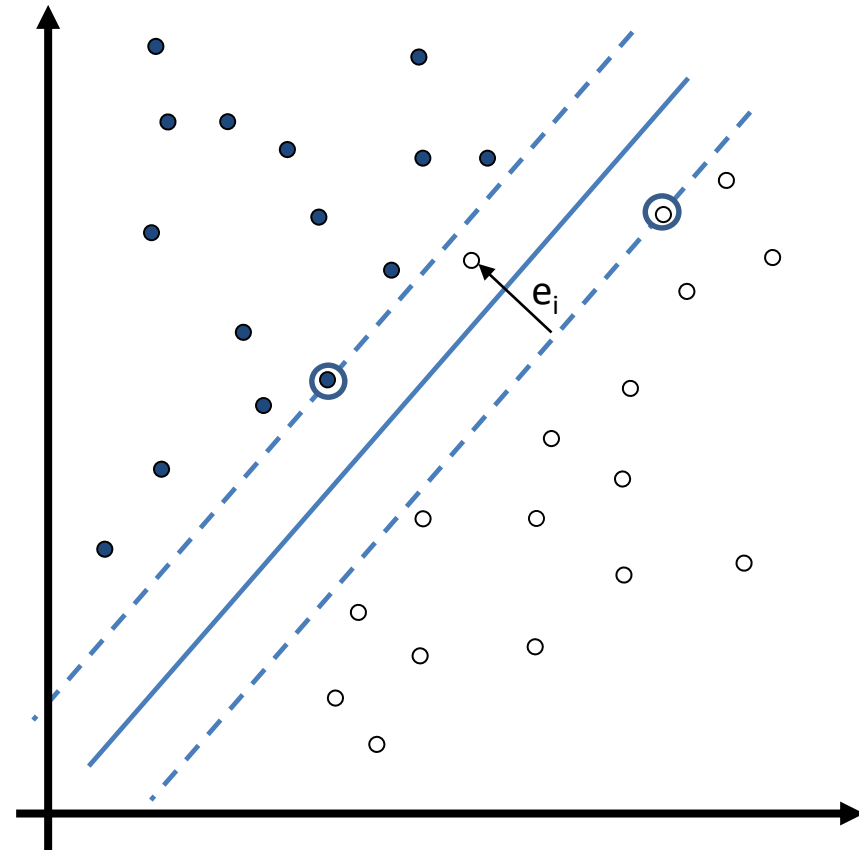
Soft Margin Formulation

Find \mathbf{w} and b such that

$\frac{1}{2} \mathbf{w}^T \mathbf{w} + C \sum e_i$ is minimized and for all $\{(\mathbf{x}_i, y_i)\}$

$$y_i (\mathbf{w}^T \mathbf{x}_i + b) \geq 1 - e_i \quad \text{and} \quad e_i \geq 0 \text{ for all } i$$

Where C is a parameter



Increasing C leads to a harder margin