Note on Funding Value Adjustments

(Preliminary Draft)

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This note is a further simplification of the simplest model in Leif Andersen, Darrell Duffie and Yang Song (2016), Funding Value Adjustments, `http://ssrn.com/abstract=2746010`

We work with the simplest possible assumptions:

- Two period model: the derivative transaction is entered into in period 0 and the payoff happens in period 1.
- The derivative is a one period swap (or more simply a forward contract) whose payoff is $X - K$ where X is the "floating rate" and K is a constant "fixed rate".
- Cash inflows are used to retire (unsecured) debt and cash outflows are met by fresh (unsecured) borrowing (symmetric Funding Value Adjustments).
- An infinitesimal amount of the swap is bought so that the transaction does not change the default probability and credit spread.
- The counterparty is default free so that we can ignore the CVA adjustment and focus on DVA and FVA.
- All calculations are made under the risk neutral measure.
- The risk free rate is zero.
- Default and recovery are independent of X
- Derivative counterparty ranks pari passu with other (unsecured) creditors of the firm.

The credit spread is given by $s = \dfrac{\phi^*}{1 - \phi^*}$ where ϕ^* is the expected default loss rate:

$$\phi^* = E\left[\frac{L - kA}{L} 1_D\right] = PD \times LGD$$

where L is the total liabilities and A is the total assets, k is the fraction of assets that are recovered under default, and 1_D is the indicator function of the default event. PD is the risk neutral probability of default and LGD is the risk neutral expectation of the loss given default. Note that we can go in the other direction as well: $\phi^* = \frac{s}{1+s}$.

From the point of view of the firm (or of an investor who owns a vertical slice of the capital structure: unsecured debt and equity), the value of the swap is:

$$v = E[X - K] + \phi^* E[(X - k)^-]$$

The first term is the value of the swap ignoring default risk and the second term is the DVA which reflects the fact that at default, the derivative counterparty absorbs part of the default loss and this is a gain to the unsecured creditors of the firm. Recall that we are ignoring CVA by assuming that the counterparty is default free.

Since shareholders care only for cash flows in the non default state, the value v^* of the swap to the shareholders is given by:

$$E[X - K] = v^*(1 + s) \text{ or } v^* = \frac{E[X-K]}{1+s}$$

The expectations in the above expressions should actually be the conditional expectation given no default, but under the independence assumption, this has been replaced by the unconditional expectation.

We now compute v and v^* and the Funding Value Adjustment $(v - v^*)$ for two special cases where X is actually constant:

- $X = 1$ and $K = 0$ gives us $v = 1$ and $v^* = \frac{1}{1+s}$. Under our assumption that cash outflows are financed by borrowing, this swap is like borrowing to invest in the risk free asset. The Funding Value Adjustment $(v - v^*)$ in this case merely reflects the fact that borrowing money and investing in the risk free asset (at its fair value of v) is a transfer of wealth from shareholders to pre-existing creditors:
  - At default (which happens with probability PD), the pre-existing creditors pay only $(1 - LGD)(1 + s)$ to the new lender and receive 1 from the risk free asset. The expected gain to the unsecured creditors is therefore:
    $PD(1 - (1 - LGD)(1 + s))$
    $= PD(LGD - s + sLGD)$
    $= \phi(1 + s) - PDs$
    $= s - PDs = (1 - PD)s$
  - If there is no default (which happens with probability $1 - PD$), the shareholders pay $1 + s$ to the new lender but collect only 1 from the risk free asset. The expected loss to them is $(1 - PD)s$ which is the same as the expected gain to the pre-existing creditors.

  If the transaction is done at v^* then the shareholders are indifferent, the pre-existing creditors gain a benefit and the derivative counterparty suffers a loss equal to $v - v^* = \frac{s}{1+s} = \phi^*$. Andersen, Duffie and Song argue that counterparties might be willing to do this for the same reason that they are willing to pay a bid-ask spread.

- $X = 0$ and $K = 1$ gives us $v = v^* = \frac{-1}{1+s}$. Under our assumption that cash inflows are used to retire unsecured debt, this swap is like a new borrowing which is used to repay existing borrowing. There is no transfer of wealth between shareholders and creditors and no Funding Value Adjustment.

This suggests that in the general case we break up $X - K$ into $(X - K)^+ - (X - K)^-$. The second term does not induce any Funding Value Adjustment while $(X - K)^+$ requires a Funding Value Adjustment similar to a risk free loan: $v - v^* = \phi^* E[(X - K)^+]$.

Andersen, Duffie and Song correctly argue that (unlike CVA and DVA) the FVA is purely a transfer of wealth from shareholders to pre-existing creditors and is not an adjustment that should be made to the carrying value of the derivative in the books of the firm.